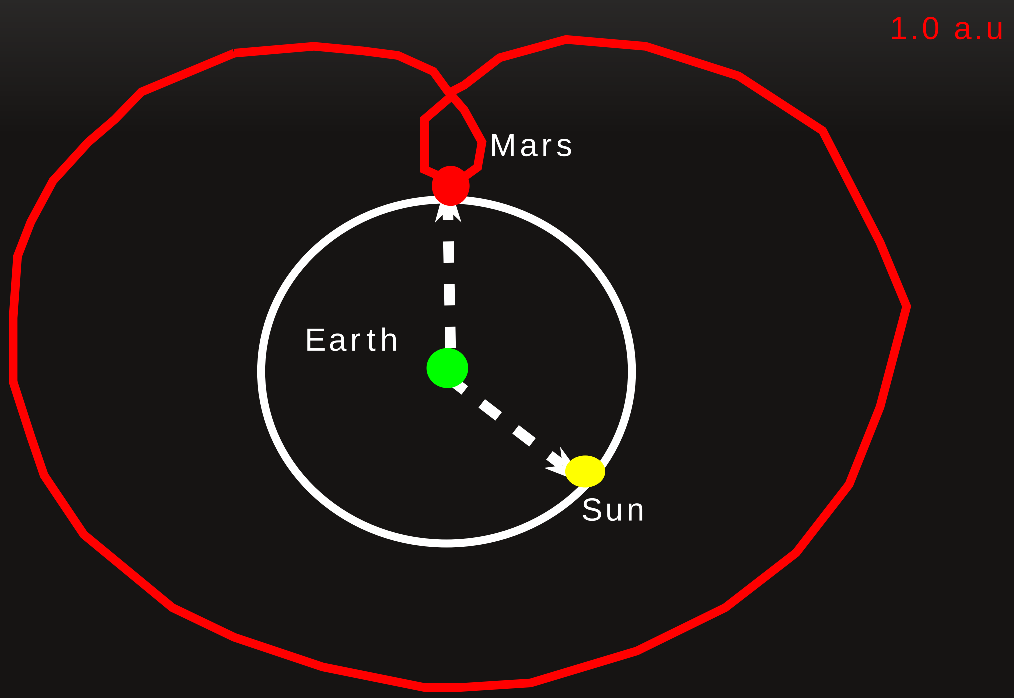
Ptolemy-Copernican Debate - Is the Earth or the Sun the center of solar system?

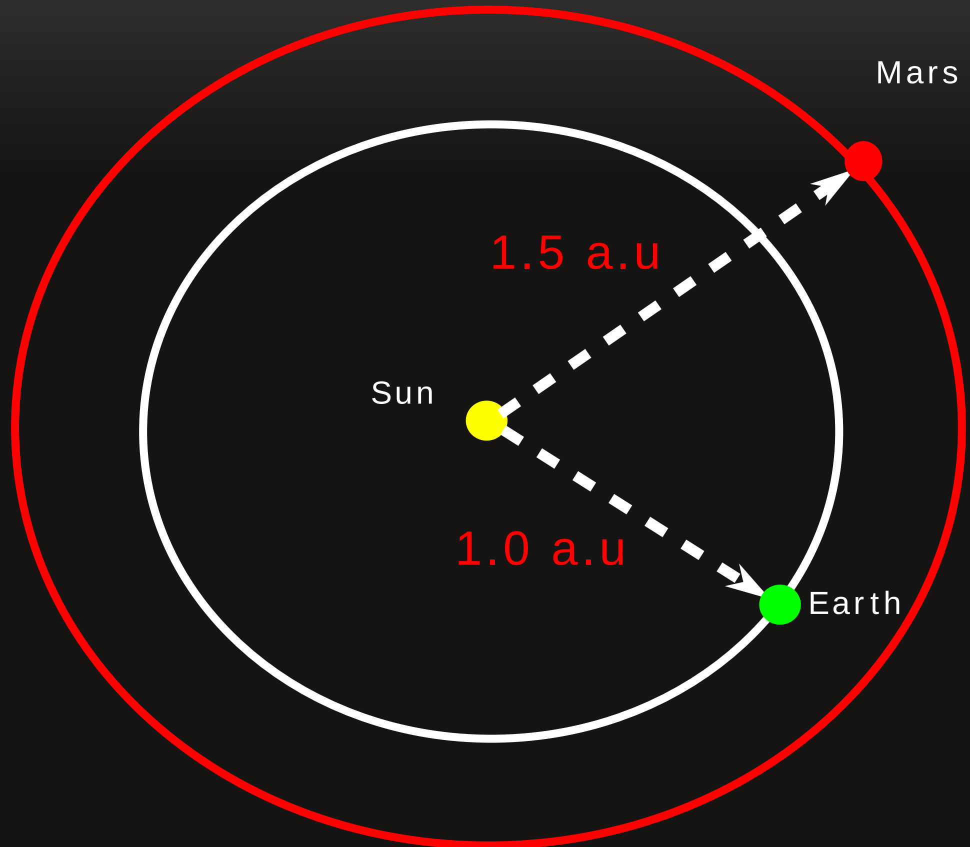
Ver. 2.2

In 1600 C.E., educated but reasonable minds disagreed as to whether the Earth or the Sun was the center of the Solar System. Trignometry played a role in the debate by showing that the apparent motion of the planets is explained in models by the same equations.



Earth-centered Ptolemy

Fig. 1



Sun-centered Copernicus

Fig. 2

Ptolemty-Copernican Debate - The best minds of 1600 C.E. disagreed on . . .

Earth-centered Ptolemy

Earth at center doesnot rotate. The Sun revolves around Earth. Based on common sense, people were unwilling to accept that the Earth's surface is moving at 1,500km.

The model is inconsistent with observed motions of Venus and Mercury.

The accuracy of predicted positions of planets is moderate. The model needs "antigravity" to work.

Sun-centered Coperincus

The Earth rotates and revolves around Sun.

The model is consistent with observed motions of Venus and Mercury.

The accuracy of predicted positions of planets is better. The model does not need "anti-gravity" to work.

Ptolemy-Copernican debate - Apparent retrograde motion of the planets

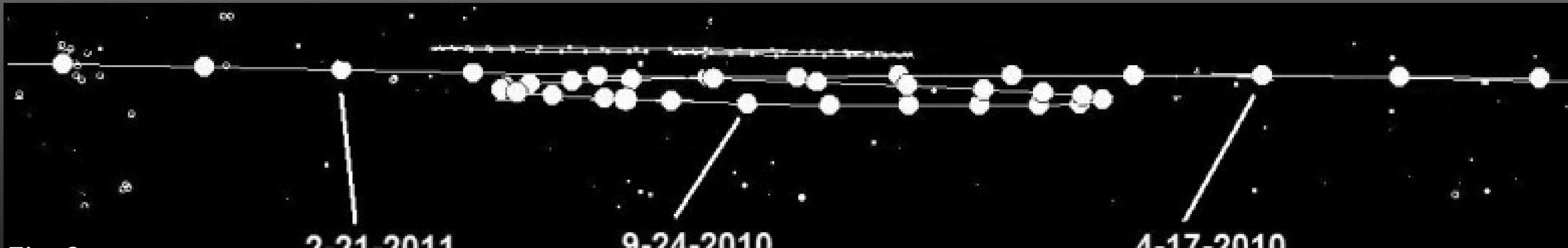


Fig. 3

Apparent retrograde motion of Jupiter Nov. 2010 as seen from Salt Lake City

Prolate cycloid

Fig. 4 - Apollonios of Perga

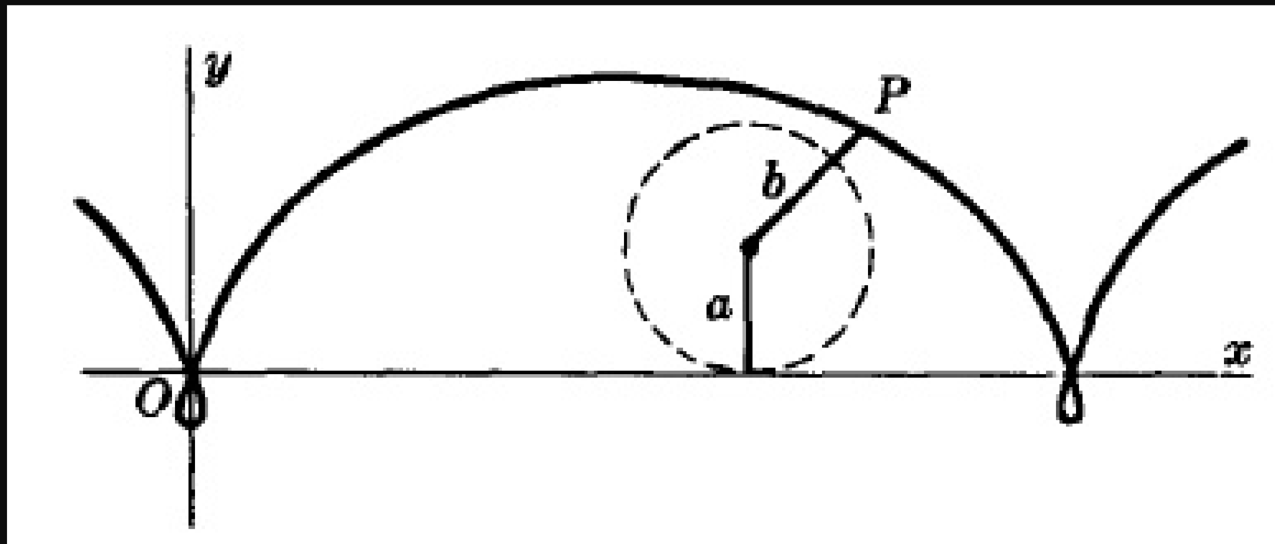


Fig. 5 - Prolate cycloid

See LHP, Ex.s 10.6.45, .47 and .50, p. 777; LHP 775

Ptolemy modeled planet positions using a parametric eq. for a prolate epitrichoid

$$x = d\left((a+b)\cdot\cos(t) - h\cdot\cos\left[\frac{(a+b)\cdot t}{b}\right]\right)$$

$$y = d\left((a+b)\cdot\sin(t) - h\cdot\sin\left[\frac{(a+b)\cdot t}{b}\right]\right)$$

Fig. 6 - Prolate epitrichoid - parametric equation. Source: Eq. 3.18.5(3). CRC Standard Curves (1993)

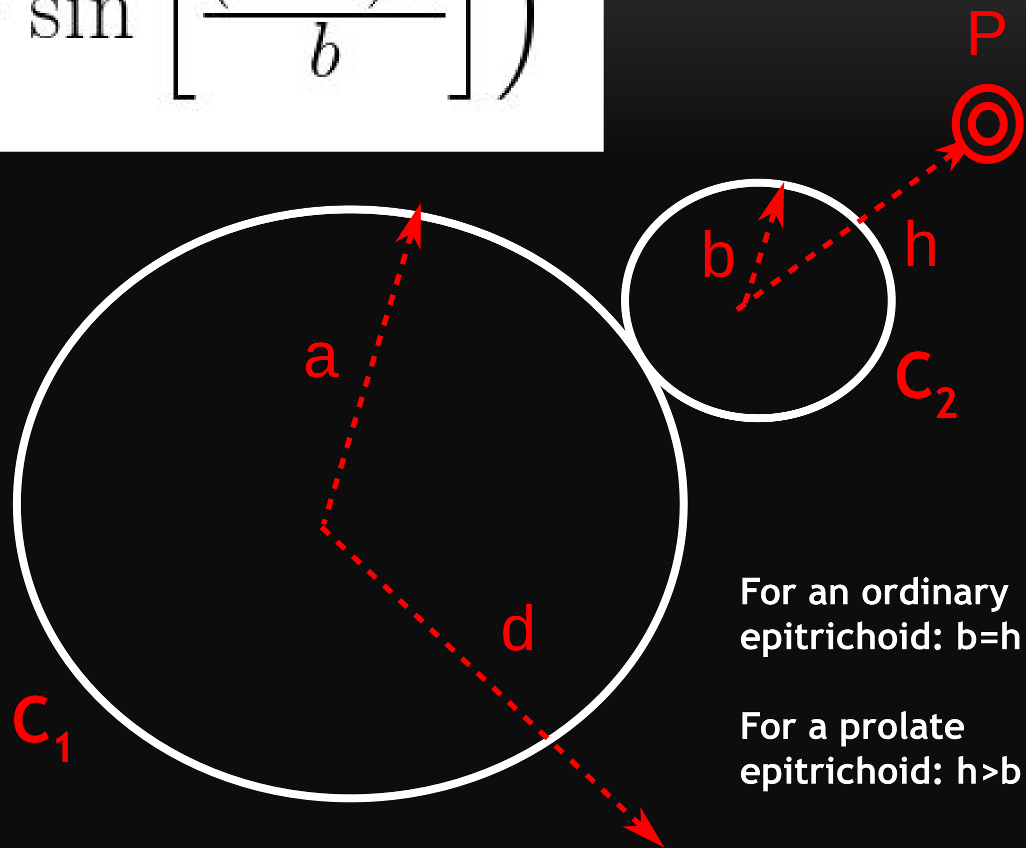


Fig. 7 - Prolate epitrichoid - key parameters

For an ordinary epitrichoid: b=h

For a prolate epitrichoid: h>b

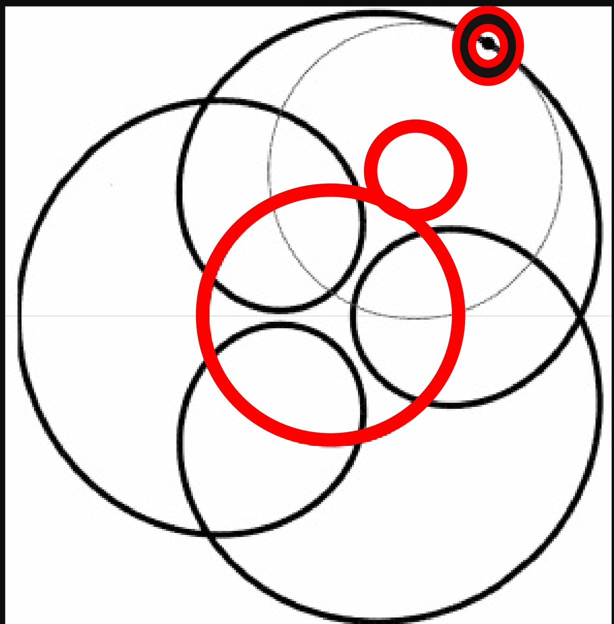


Fig. 8 - Prolate epitrichoid - Modified Ex. 6.18. Modern Differential Geometry (3rd) (2006)

Data needed to find the constants for the prolate epitrichoid model of the orbit of Mars

Body	Dist. to Earth (a.u.)	Synodic P (y) Angular speed (degs/day)	Tropical P (y) Angular speed (degs/day)
Sun	1.0	1 (0.985 deg/d)	1 (0.985 deg/d)
Mars	1.5	2 (0.461 deg/d)	1.88 (0.524 deg/d)

Mars "Radius of the Epicycle": 0.65630

Source: Evans (1998, p. 363, 369).

Trignometric math that solves for the constants of orbit of Mars in Ptolemy's prolate epitrichoid model.

d = 3/2 (or 1.5) to scale a and b to Mar's orbit

h is given as 2/3 (or 0.666)

a + b = 1.0 ; (a+b)/b=2

Solve simultaneous equations
by back substitution

(a+b)/b=2 ; a+b=1 ; substitute

(1.0)/b=2

b=1/2 ; a =1/2

See LHP, Sec. 7.1,
p. 496-497 on back substitution.

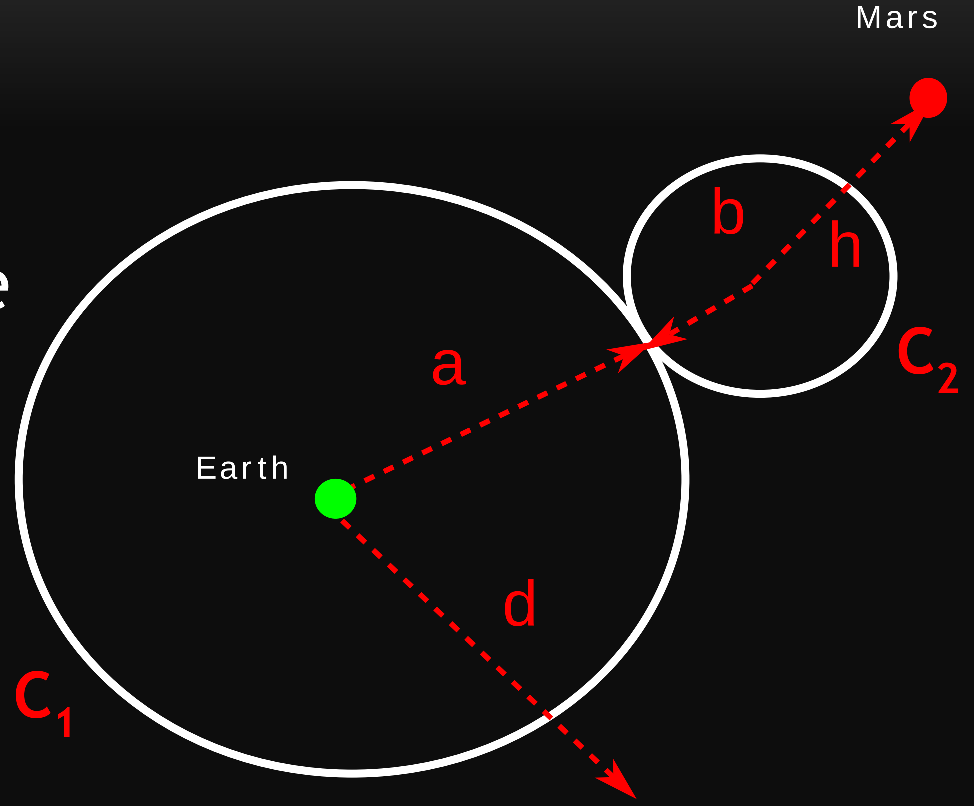


Fig. 9 - Prolate epitrichoid - key variables

And the answer is - Ptolemy's solved equation for the apparent motion of Mars in an Earth-centered model

$$x = 1.5\cos(t\,\pi) - (1)\cos(2t\,\pi)$$
$$y = 1.5\sin(t\,\pi) - (1)\sin(2t\,\pi)$$

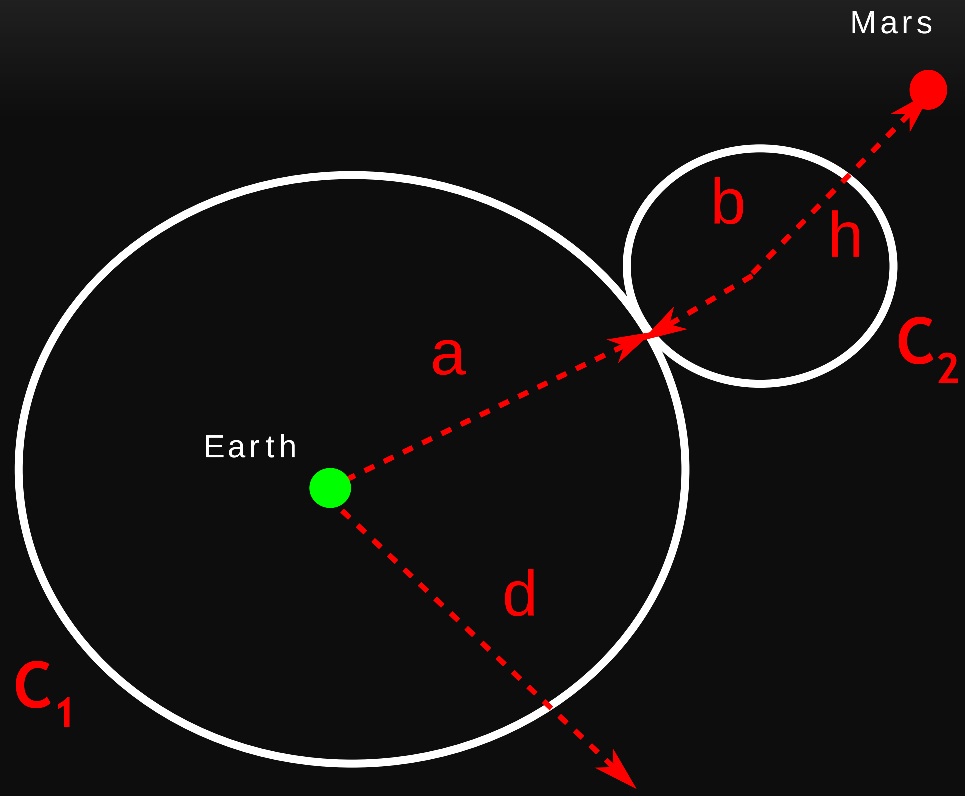


Fig. 9 - Prolate epitrichoid - key variables

Copernican view - What if we put the Sun at center?

Keplerian parametric eq. for Mars and Earth in a

Sun-centered solar system

Mars

$x = 1.5\cos(t\ pi)$

$y = 1.5\sin(t\ pi)$

Earth

$x = (1)\cos(2t\ pi)$

$y = (1)\sin(2t\ pi)$

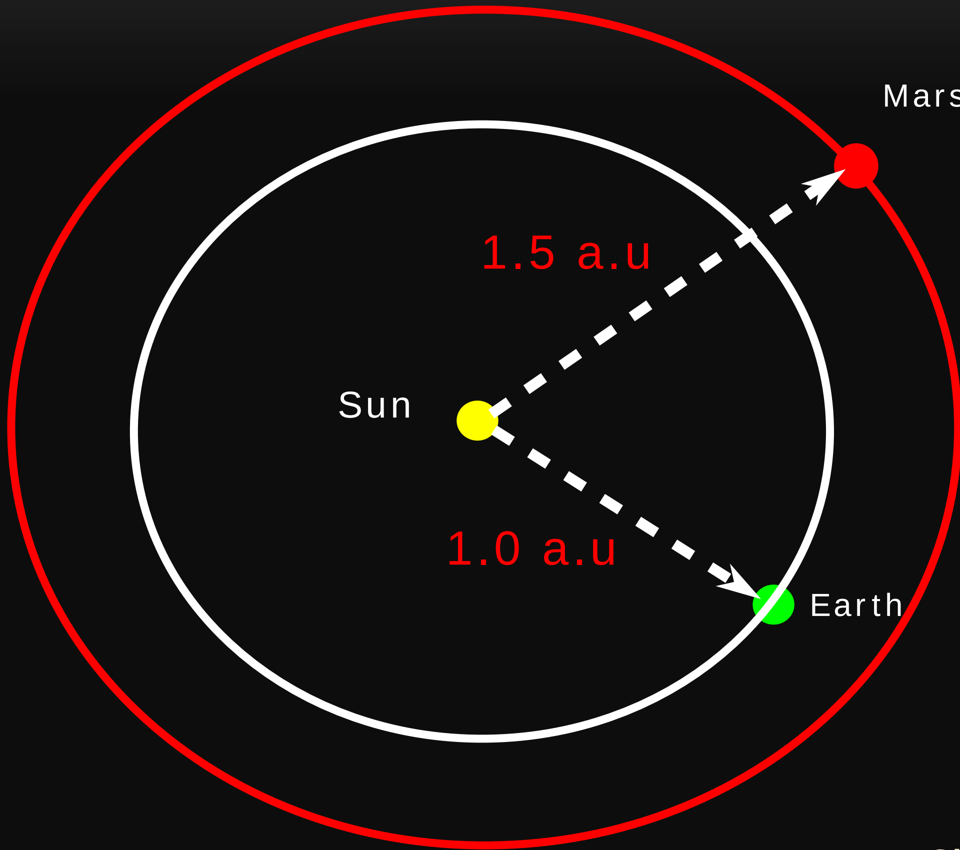


Fig. 10 - Copernican-Keplerian model

Copernican-Keplerian model of the apparent motion of Mars from the Earth

The apparent motion of Mars as seen from Earth equals the position of Mars minus the poisition of the Earth.

Plus: Mars $x = 1.5\cos(t\,pi)$ $y = 1.5\sin(t\,pi)$

Minus: Earth $y = (1)\cos(2t\,pi)$ $y = (1)\sin(2t\,pi)$

Equals:

$x = 1.5\cos(t\,pi)-(1)\cos(2t\,pi)$

$y = 1.5\sin(t\,pi)-(1)\sin(2t\,pi)$

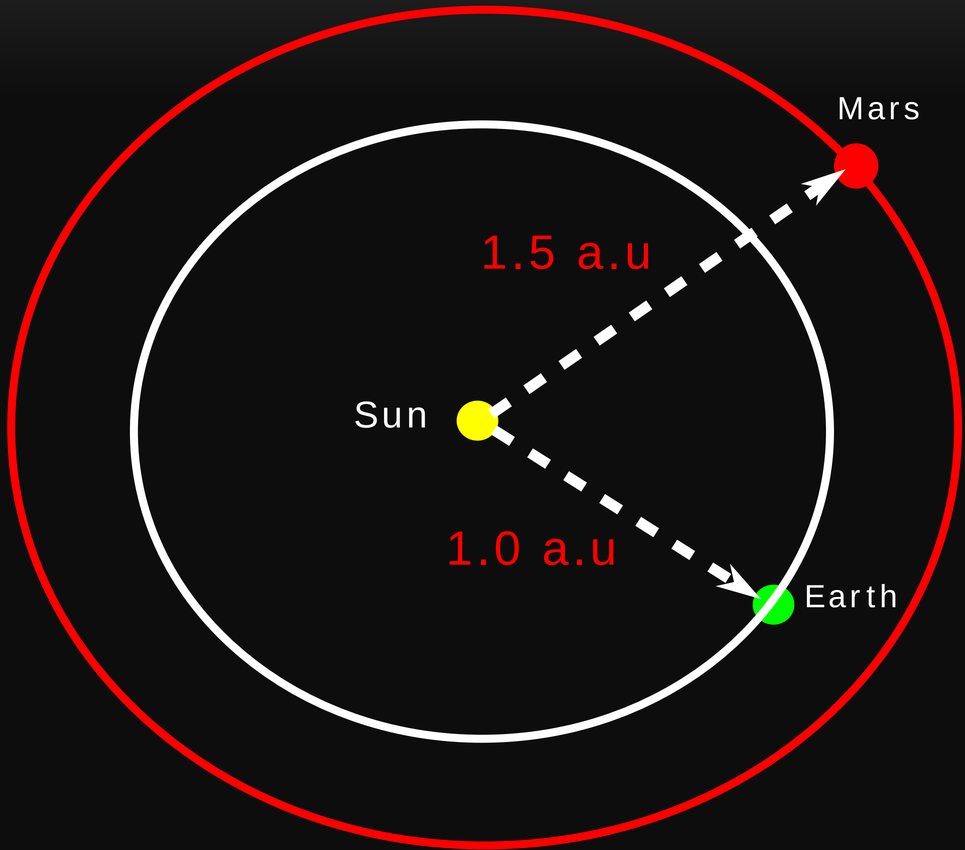


Fig. 10 - Copernican-Keplerian model

Ptolemy's Earth-centered model and Copernicus's Sun-centered model plot the same apparent motion of Mars!

Ptolemy's equation for apparent motion of Mars for an Earth-centered solar system

$$x = 1.5\cos(t\,\pi) - (1)\cos(2t\,\pi)$$

$$y = 1.5\sin(t\,\pi) - (1)\sin(2t\,\pi)$$

Keplerian equation for apparent motion of Mars for a Sun-centered solar system

$$x = 1.5\cos(t\,\pi) - (1)\cos(2t\,\pi)$$

$$y = 1.5\sin(t\,\pi) - (1)\sin(2t\,\pi)$$

Ptolemy-Copernican debate - Who won?

Although the two models agreed for the motion of Mars, they differed for the motion of Venus. In a Ptolemaic Earth-centered solar system model, Venus should only reach a crescent phase illumination, and Venus should never proceed to quarter phase illumination. In a Copernican Sun-centered solar system model, Venus goes through all phases. In 1623, Galileo used a telescope to look at the phases of Venus and saw Venus proceed to a full phase. Galileo's observations conclusively ended the debate in favor of a Sun-centered solar system. But the Ptolemaic model continued in use for the next 150 years because people were unwilling to accept that they are standing on the surface of a sphere that is rotating at 1,500 km/per hour.

That the Ptolemaic and Keplerian equations for the apparent motion of the planets plotted the same motion helped scientists to move toward the physically correct Sun-centered model.

Fig. 11 - Galileo's drawing of his telescopic observations of the phases of Venus from Il Saggiatore **(The Assayer) (1623)**

Lessons learned from the Ptolemy-Copernican debate for your future studies -

1. Mathematical models of reality are not reality. Do not confuse a mathematical model that you build with reality. Math modeling is useful to guide your intuition away from poorly formed notions about how things work.

2. At times, more than two or more equally probable mathematical models can describe the world, but you have to be diligent in looking for the one that best describes real events. Always look for inconsistencies

3. Many useful mathematical models can be constructed for use in engineering, economics, medicine, biology, and sociology by adding or subtracting two sets of trignometric equations. You will be using trignometric tools that you learned in this course and the method demonstrated here over the next four years. See LHP Sec. 10.6, 10.8; see LHP Exercise 4.6.88.

Ptolemy-Copernican debate - What is the apparent position of a planet seen from Earth?

This presentation is an abbreviated and modified version of a video lecture given by Dr. Donald G. Saari, U.C. Irvine (2010) titled "Understanding Ptolemy's Enduring Achievement." Dr. Saari is the Chief Editor of the Bulletin of the American Mathematical Society.