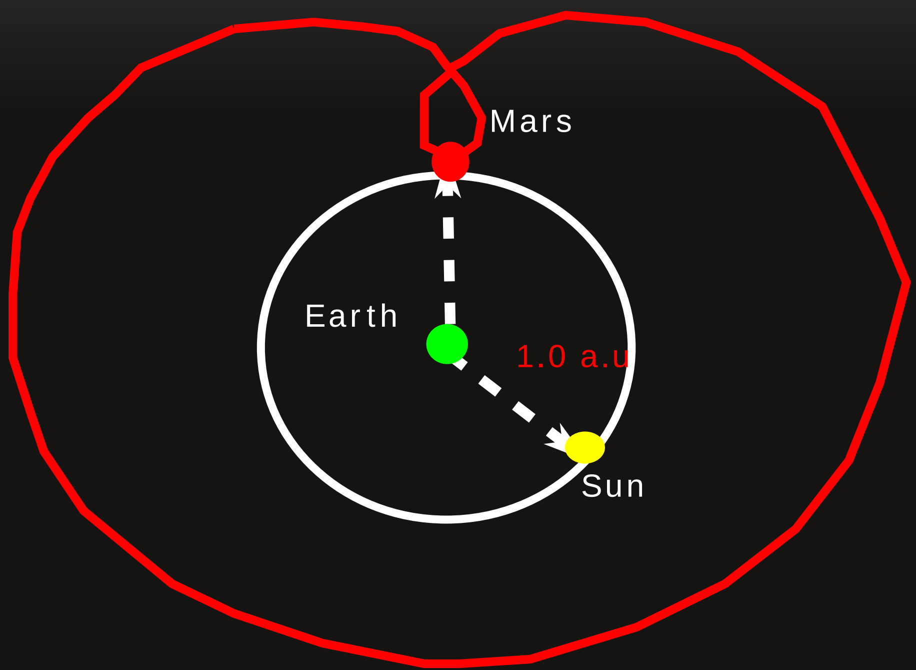
Ptolemy-Copernican Debate - Is the Earth or the Sun the center of solar system?

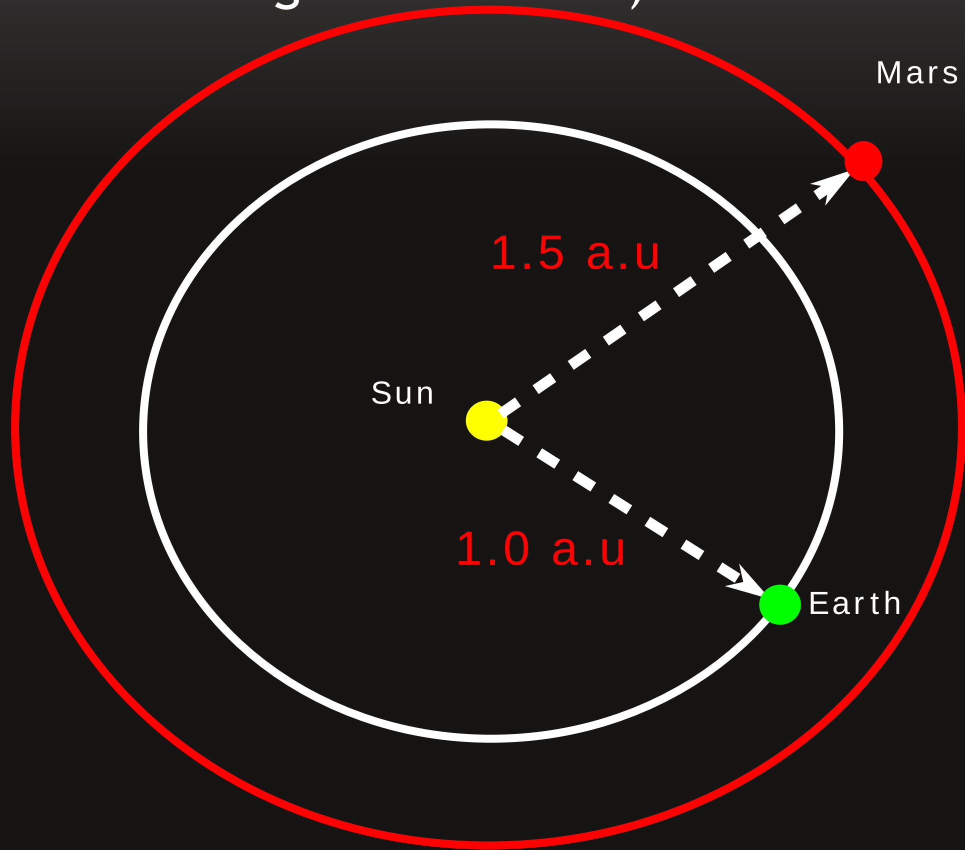
Astro Ver. 1.0

In 1600 C.E., educated but reasonable minds disagreed as to whether the Earth or the Sun was the center of the Solar System. Trignometry played a role in the debate by showing that the apparent motion of the planets is explained in models by the same equations. (Note Sun has no retrograde motion.)



Earth-centered Ptolemy

Fig. 1



Sun-centered Copernicus

Fig. 2

Ptolemy-Copernican Debate - The best minds of 1600 C.E. disagreed on . . .

Earth-centered Ptolemy

Earth at center doesnot rotate. The Sun revolves around Earth. Based on their physical senses, people were unwilling to accept that the Earth's surface is moving at 1,500km. The model is inconsistent with observed motions of Venus and Mercury. Unable to measure length of year accurately. The model better simulates the albedo effect of planets at opposition. The accuracy of predicted positions of planets is moderate. The model needs "antigravity" to work.

Sun-centered Copernicus

The Earth rotates and revolves around Sun. Requires acceptance of Newtonian physics. The model is consistent with observed motions of Venus and Mercury. Equation of time rises from solar orbit. The model does not simulate the non-linear planetary albedo effect at opposition well. The accuracy of predicted positions of planets is no significantly better. The model does not need "anti-gravity" to work.

Ptolemy-Copernican debate - Apparent retrograde motion of the planets

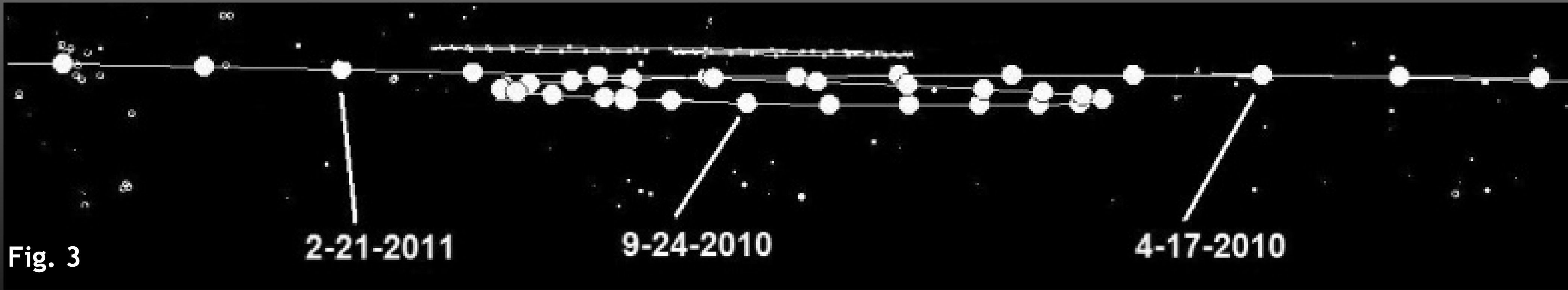


Fig. 3

Apparent retrograde motion of Jupiter Nov. 2010 as seen from Salt Lake City

Prolate cycloid

Fig. 4 - Apollonius of Perga

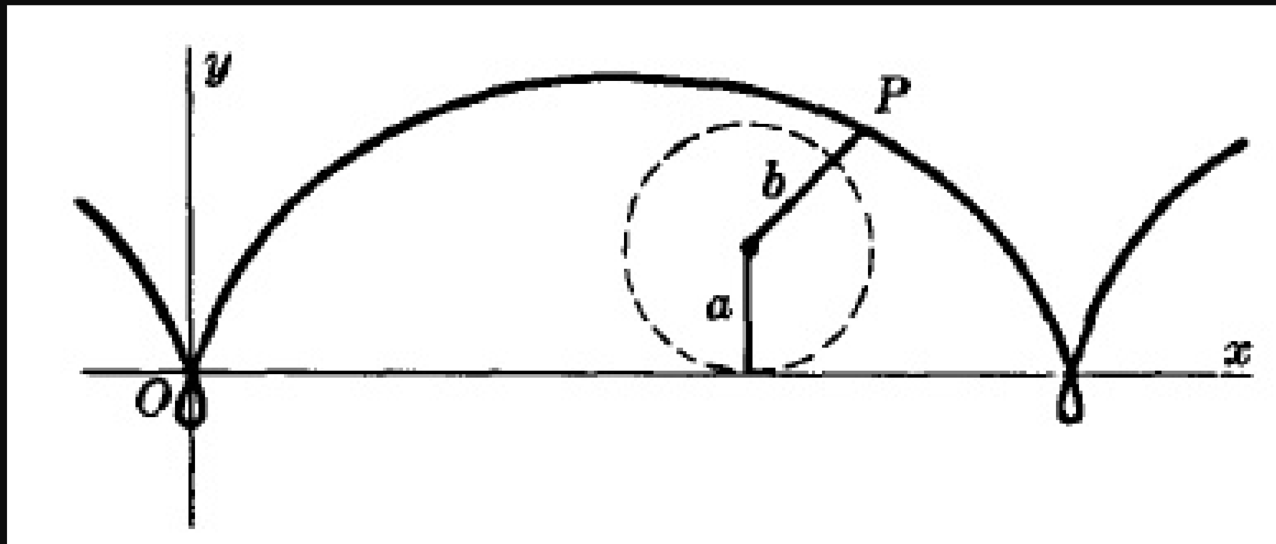


Fig. 5 - Prolate cycloid

Ptolemy-Copernican Debate - Families of epicycloids

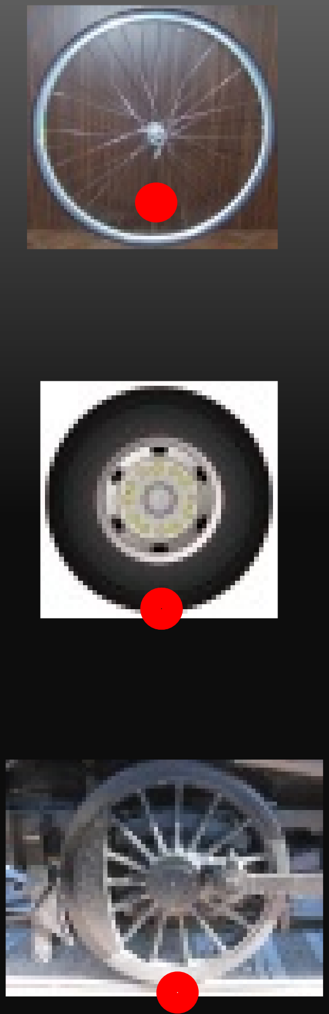


Fig.s 7a,b,c

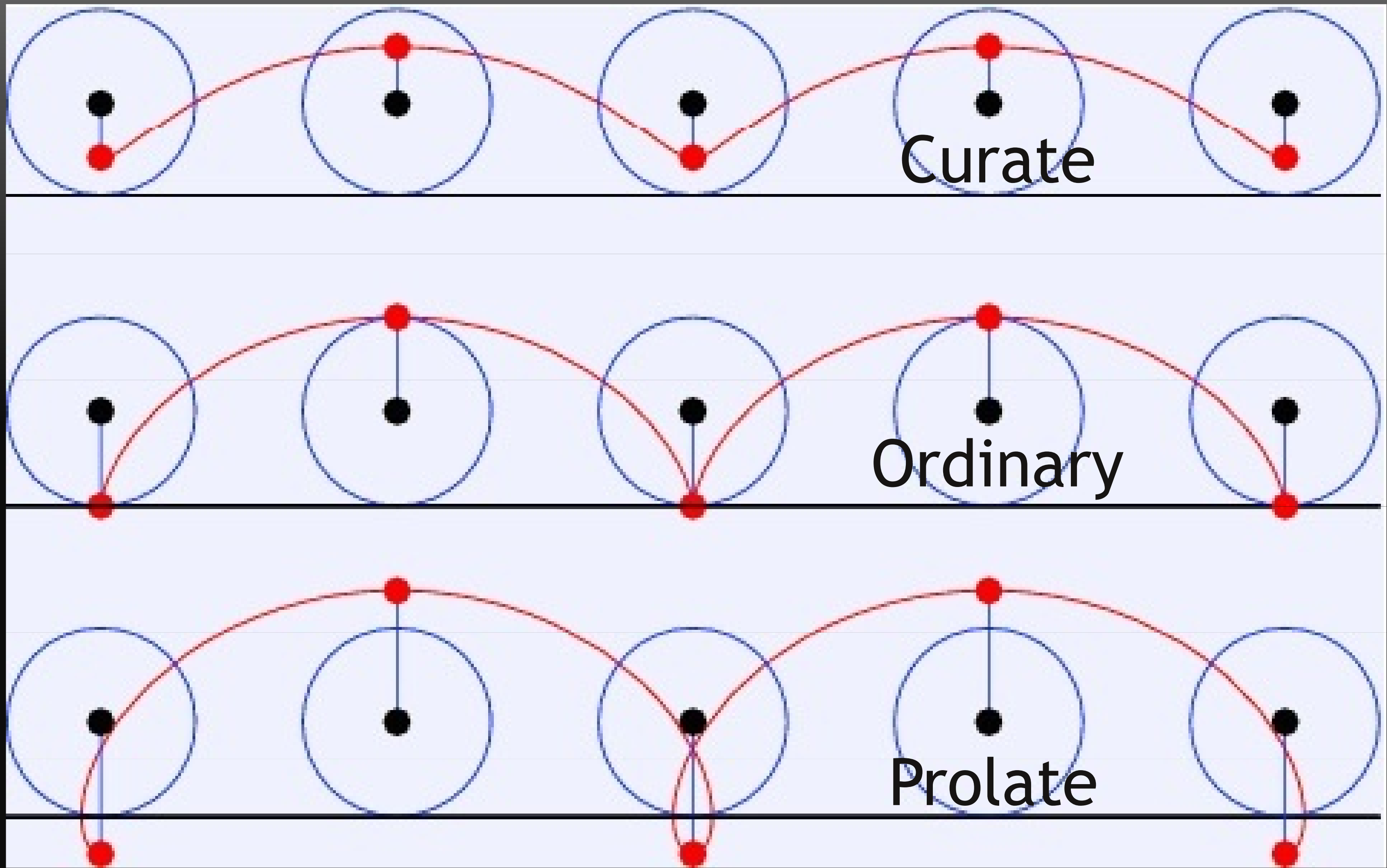


Fig. 6 - Families of epicycloids. Weisstein, Eric W. "Trochoid." From MathWorld--A Wolfram Web Resource. http://mathworld.wolfram.com/Trochoid.html

Ptolemy-Copernican Debate - Rolling a prolate cycloid around a circle gives a prolate epitrochoid

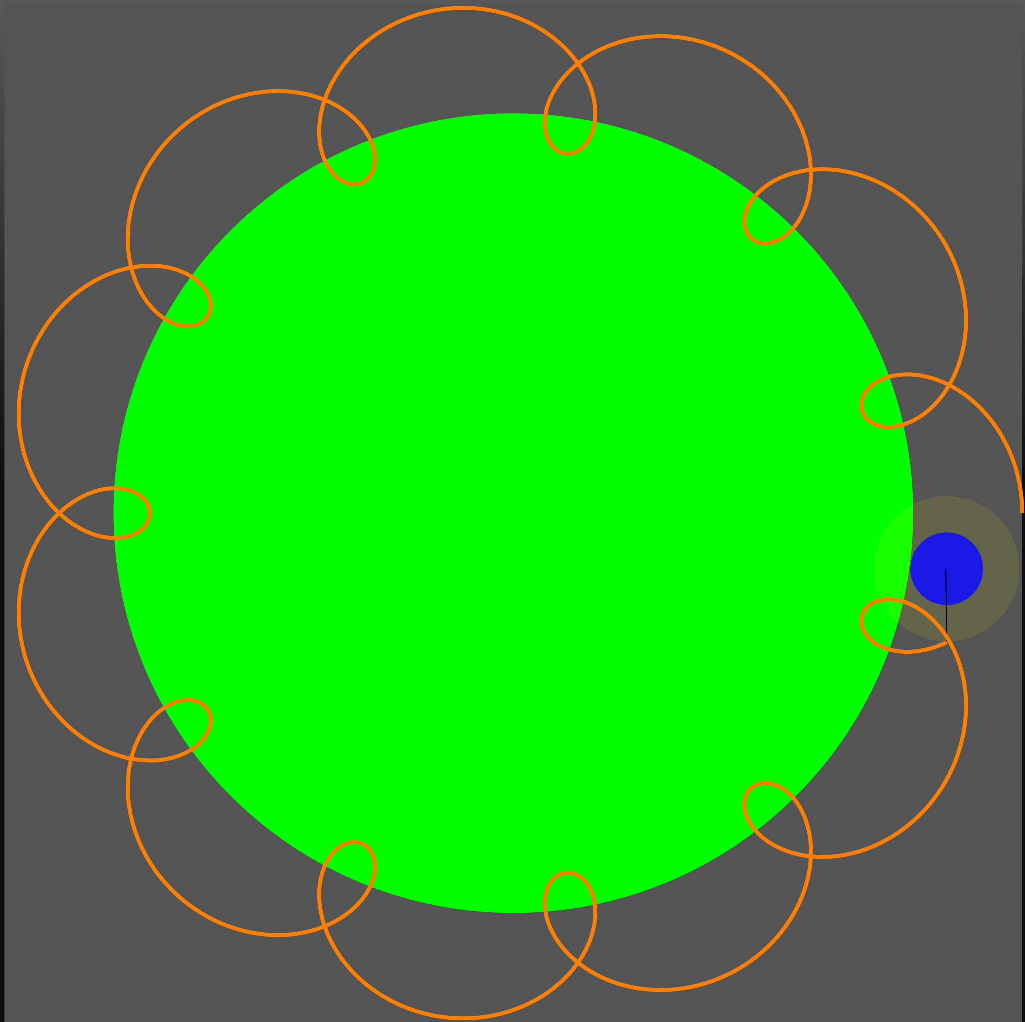


Fig. 8 - Generating a prolate epitrochoid with a rolling wheel

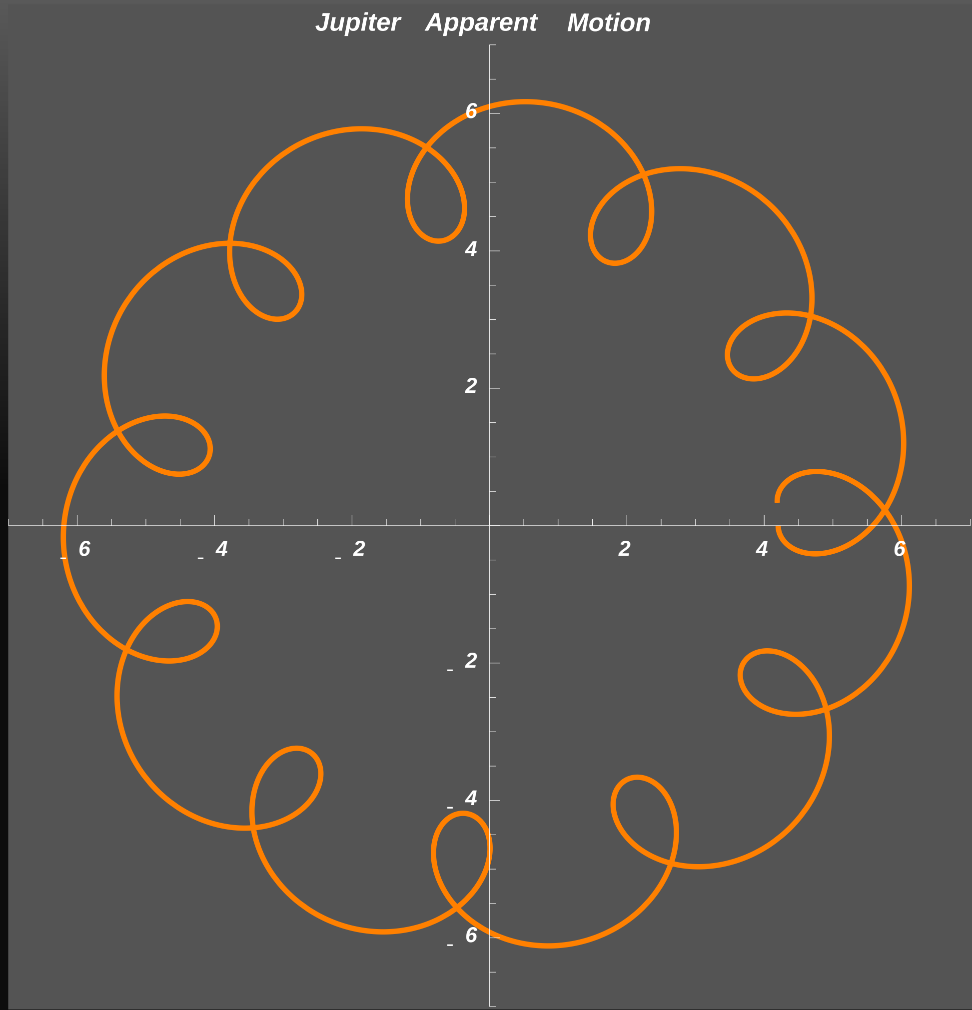


Fig. 9 - Apparent motion of Jupiter after Apollonius and Ptolemy models

Run animation Fig. 8-9

Ptolemy-Copernican Debate - What are parametric equations? Example: Plotting a circle

Rectangular circle: $r^2=x^2+y^2$

Polar circle: r(theta)=a

Parametric:

x=r*cos(t*pi)
y=r*sin(t*pi)
where 0<t<2

Key point: x and y change uniformly based on t

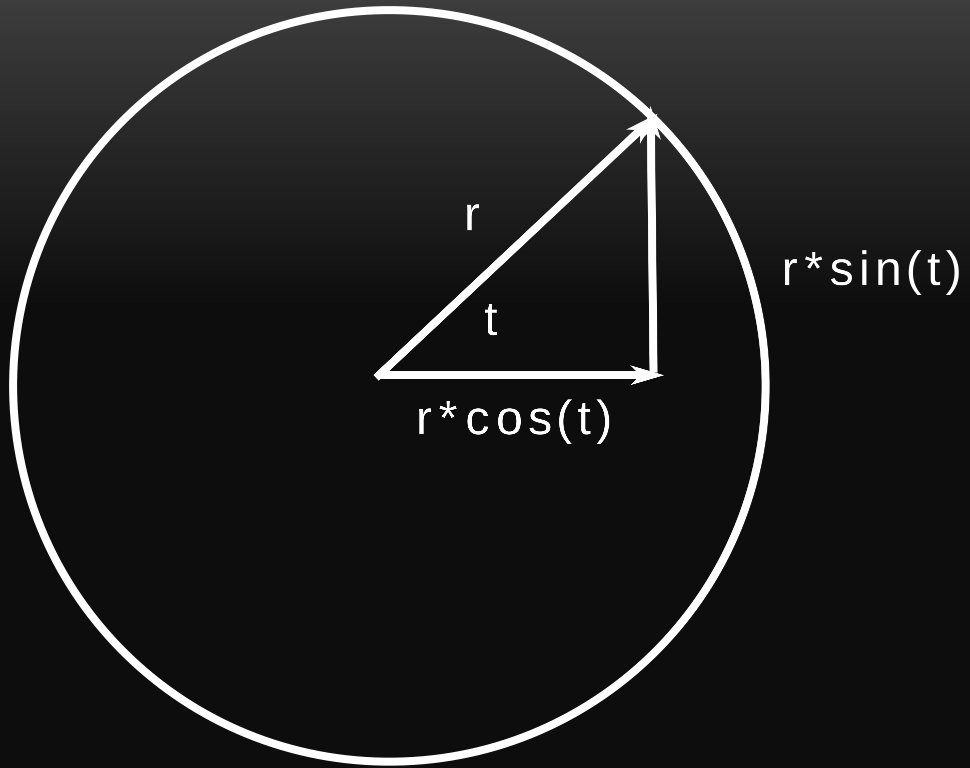
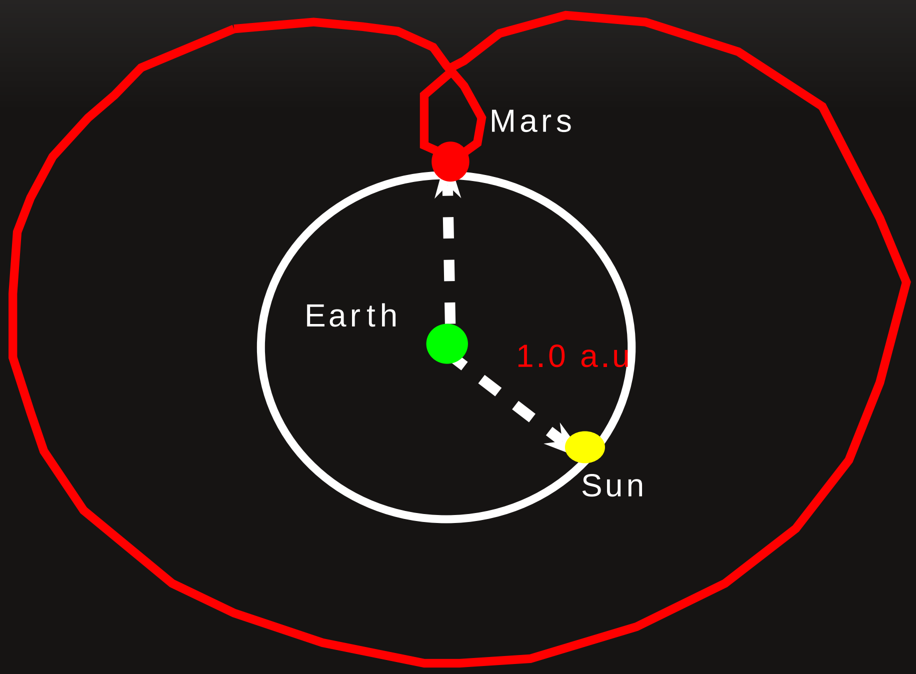


Fig. 10 - Circle - parametric equation form

Ptolemy-Cpernican Debate - A simplied Apollonius model with convenient values

To explore the Ptolemaic model of the solar system, we will use the simplified version of Apollonius without Ptolemy's equant (an off-centered deferent) and rounded values for ease of computation.



Earth-centered Ptolemy

Fig. 1

We will explore examples involving only three outer planets: Mars and Jupiter, and in the Ptolemaic model - the Sun.

Ptolemy-Copernican Debate - Plotting a prolate epitrochoid

$$x = d\left((a+b)\cdot\cos(t) - h\cdot\cos\left[\frac{(a+b)\cdot t}{b}\right]\right)$$

$$y = d\left((a+b)\cdot\sin(t) - h\cdot\sin\left[\frac{(a+b)\cdot t}{b}\right]\right)$$

Fig.11 - Prolate epitrochoid - parametric equation.
Source: Eq. 3.18.5(3). CRC Standard Curves (1993)

Run animated Fig. 12.

Fig. 12 - Graph of a prolate epitrochoid

Ptolemy modeled planet positions using prolate epitrochoids

$$x = d\left((a+b)\cdot\cos(t) - h\cdot\cos\left[\frac{(a+b)\cdot t}{b}\right]\right)$$

$$y = d\left((a+b)\cdot\sin(t) - h\cdot\sin\left[\frac{(a+b)\cdot t}{b}\right]\right)$$

Fig. 12 - Prolate epitrochoid - parametric equation.
Source: Eq. 3.18.5(3). CRC Standard Curves (1993)

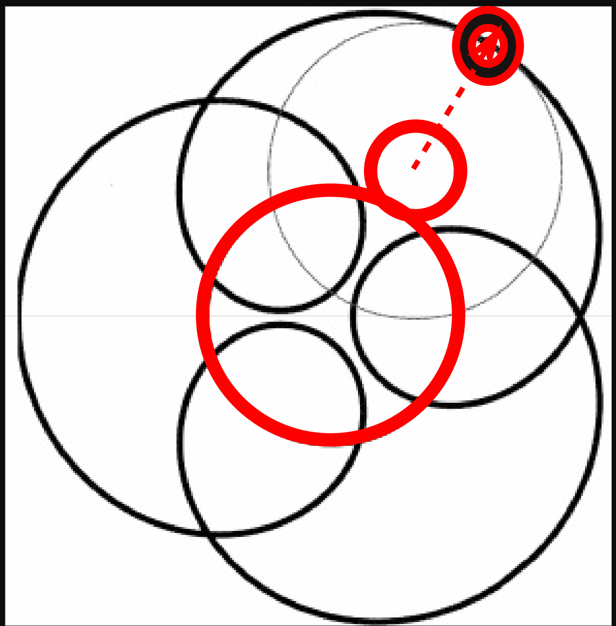


Fig. 13 - Prolate epitrochoid - Modified Ex. 6.18. Modern Differential Geometry (3rd) (2006)

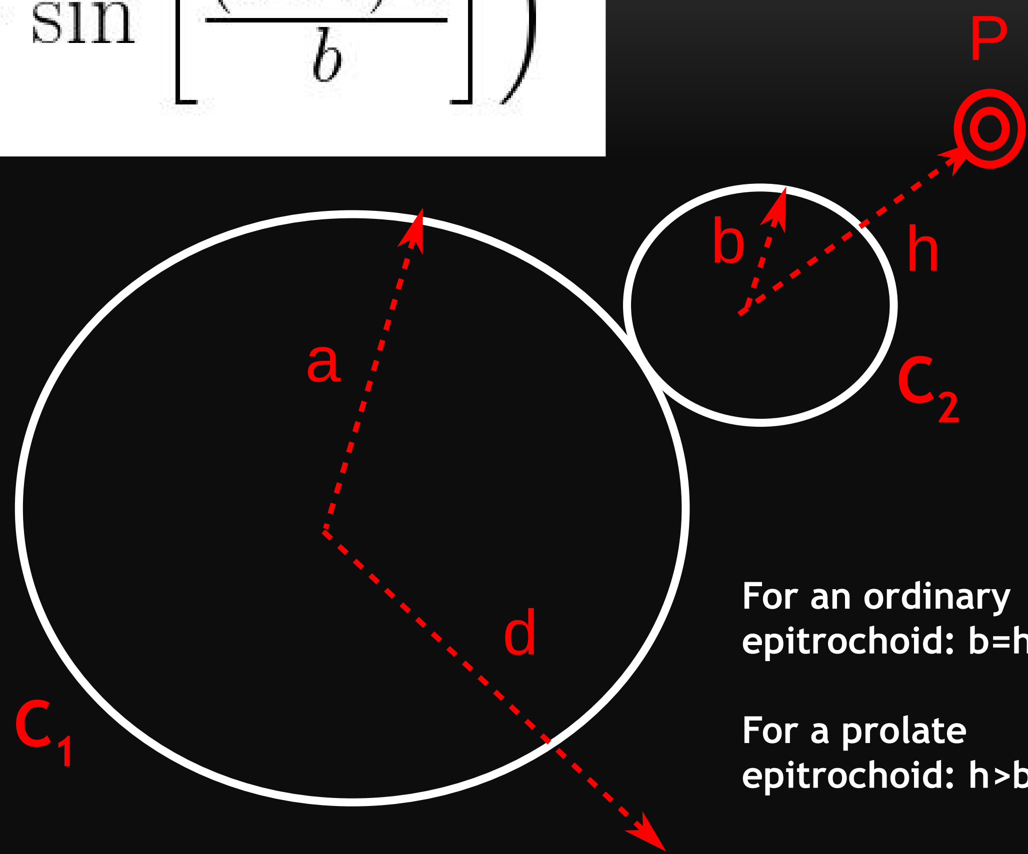


Fig.14 - Prolate epitrochoid - key parameters

For an ordinary epitrochoid: b=h

For a prolate epitrochoid: h>b

Ptolemy-Copernican Debate - Relating prolate epitrochoid constants to planetary orbits

$$x = d\left((a+b)\cdot\cos(t) - h\cdot\cos\left[\frac{(a+b)\cdot t}{b}\right]\right)$$

$$y = d\left((a+b)\cdot\sin(t) - h\cdot\sin\left[\frac{(a+b)\cdot t}{b}\right]\right)$$

Fig. 12 - Prolate epitrochoid - parametric equation.
Source: Eq. 3.18.5(3). CRC Standard Curves (1993)

a+b = 1.0

t =[0 . . 2pi]

(a+b)/b = synodic period

d = distance to planet a.u.

h = radius of the epicycle or 1/d

Data needed to find the constants for the prolate epitrochoid model of the orbit of Mars

Body	Dist. to Earth (a.u.)	Synodic P (y) Angular speed (degs/day)	Tropical P (y) Angular speed (degs/day)
Sun	1.0	1 (0.985 deg/d)	1 (0.985 deg/d)
Mars	1.5	2 (0.461 deg/d)	1.88 (0.524 deg/d)

Mars "Radius of the Epicycle": 0.65630

Source: Evans (1998, p. 363, 369).

Trignometric math that solves for the constants of orbit of Mars in Ptolemy's prolate epitrochoid model

d = 3/2 (or 1.5) to scale a and b to Mar's orbit

h is given as 2/3 (or 0.666)

a + b = 1.0 ; (a+b)/b=2

Solve by back substitution

(a+b)/b=2 ; a+b=1 ; substitute

(1.0)/b=2

b=1/2 ; a =1/2

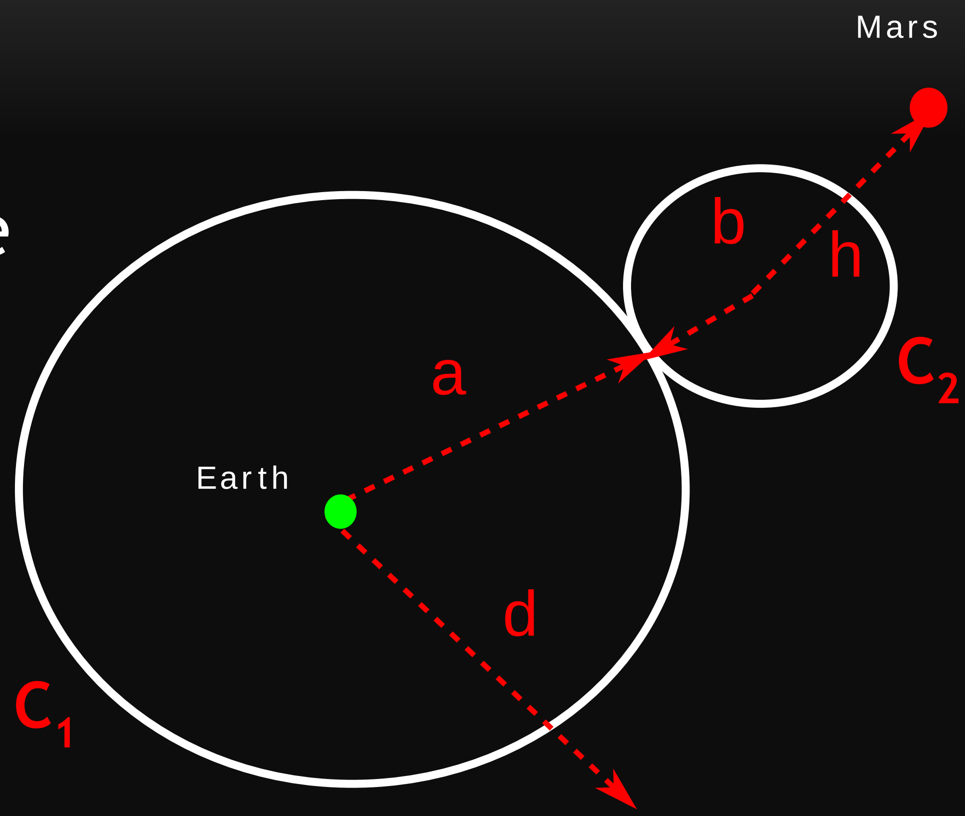


Fig. 15 - Prolate epitrochoid - key variables

And the answer is - Ptolemy's solved equation for the apparent motion of Mars in an Earth-centered model

$$x = 1.5\cos(t\pi) - (1)\cos(2t\pi)$$
$$y = 1.5\sin(t\pi) - (1)\sin(2t\pi)$$

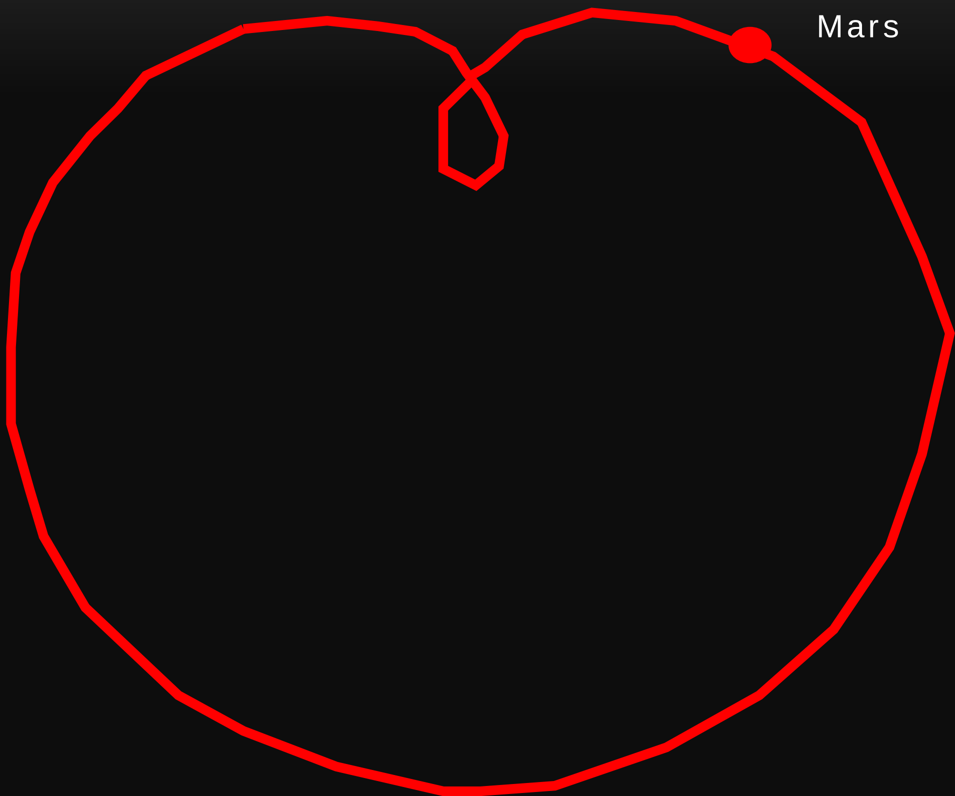


Fig. 12 - Graph of a prolate epitrochoid

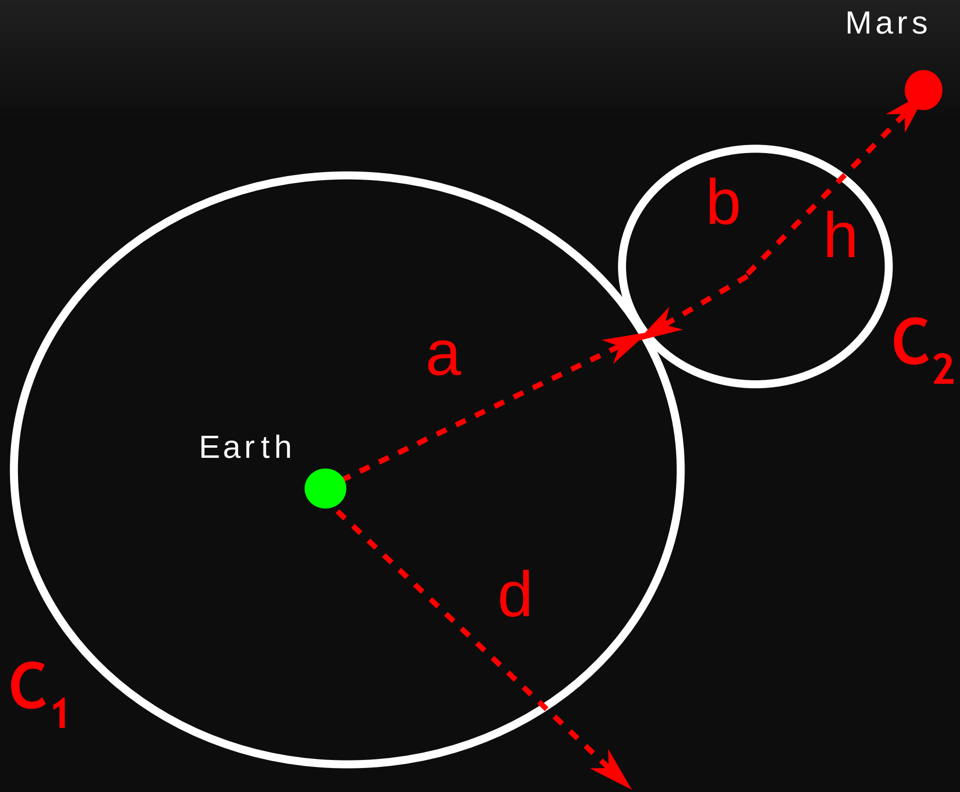


Fig. 15 - Prolate epitrochoid - key variables

Ptolemy-Copernican Debate - Ptolemy's practical method of computing solutions - Ptolemaic slats

Ptolemaic slates are a slide rule for computing the apparent positions of the planets.

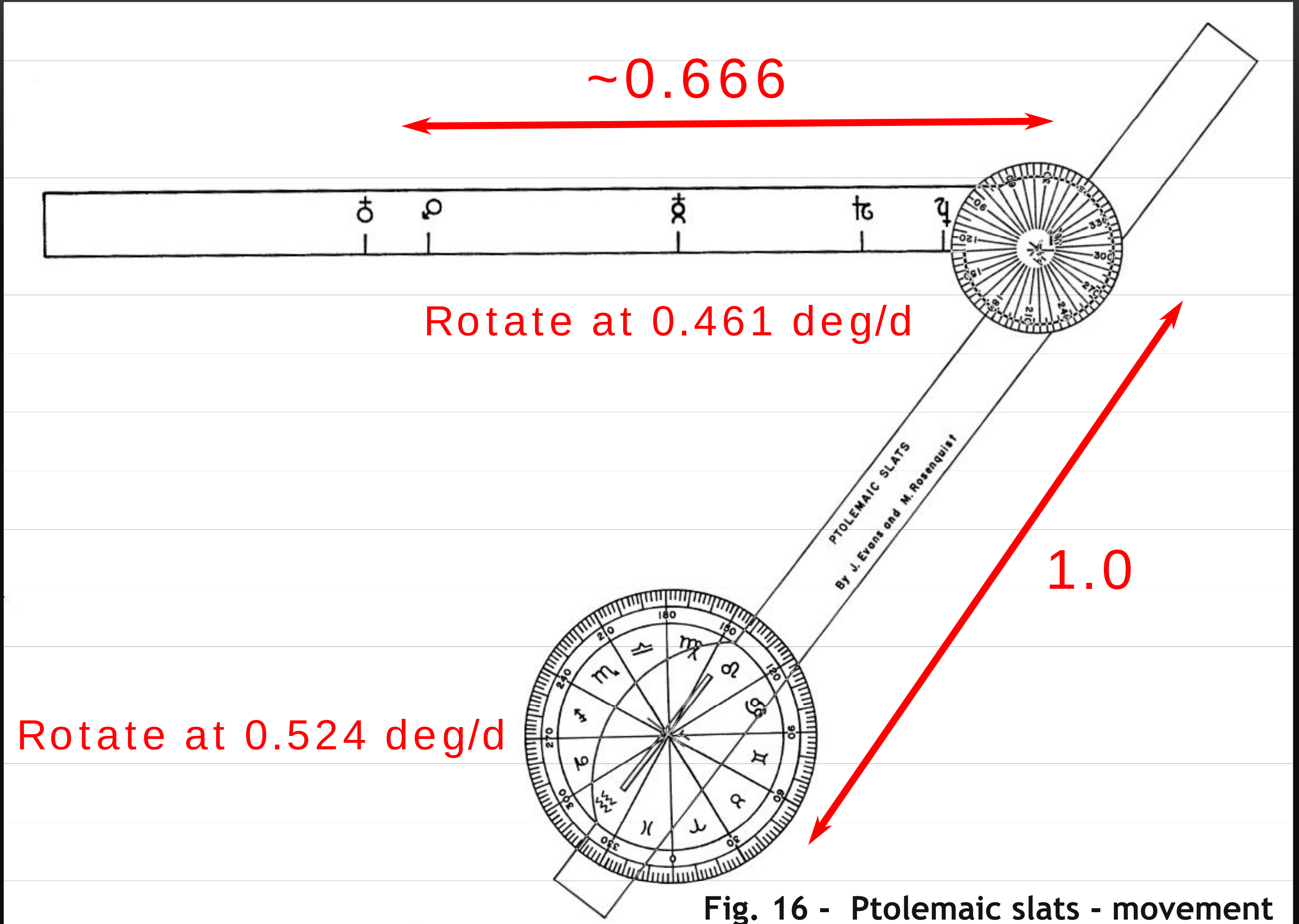


Fig. 16 - Ptolemaic slats - movement

Ptolemy-Copernican Debate - Ptolemaic slates

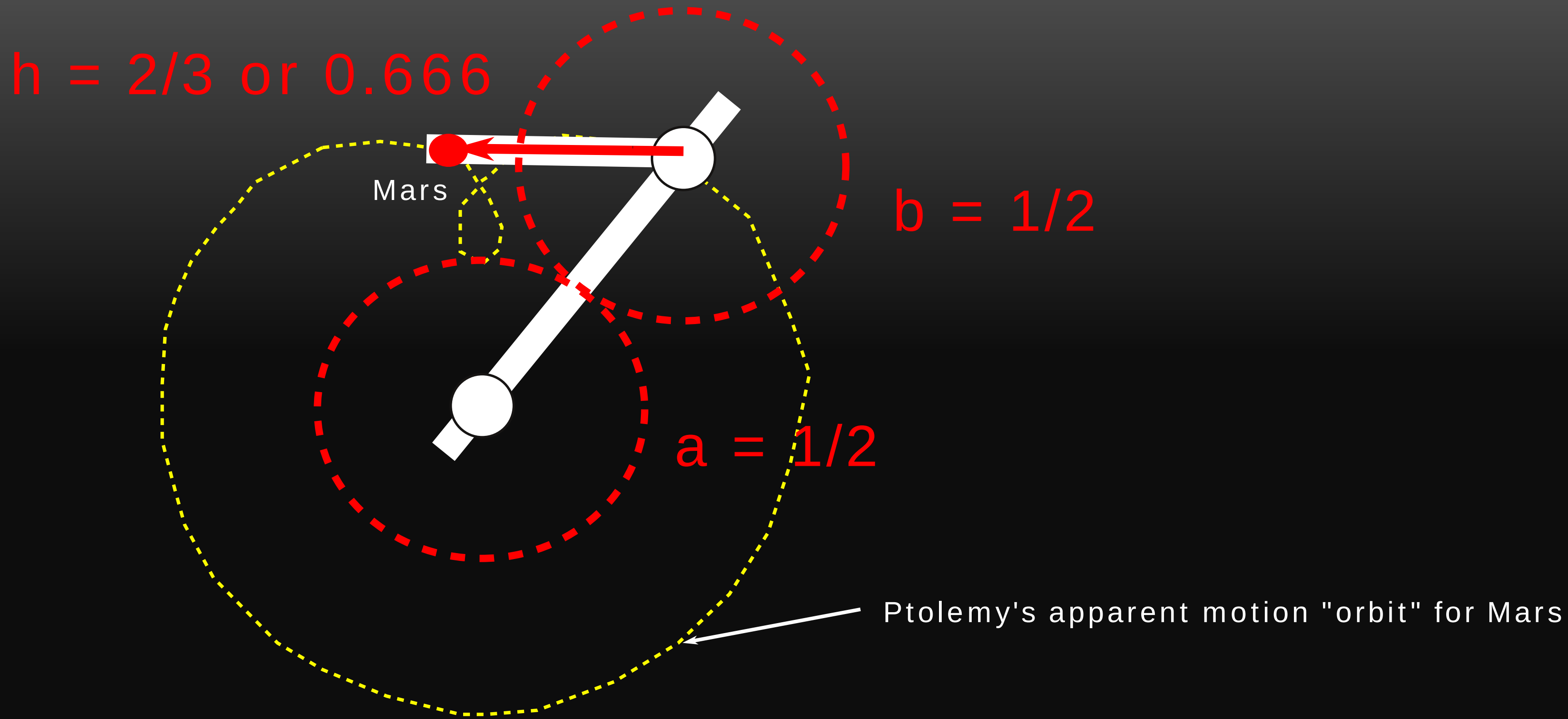


Fig. 17 - Ptolemaic slats - relationship to prolate epitrochoid and Ptolemy's apparent motion orbit of Mars

Mars "Radius of the Epicycle": 0.65630. Source: Evans (1998, p. 363, 369).

Ptolemy-Copernican Debate - generalized prolate epitrochoid formula adjusted to Ptolemaic planetary orbits

$$x = d \cdot [\cos((n \cdot \delta))] - \cos((b \cdot (n \cdot \gamma)) + \nu))$$
$$y = d \cdot [\sin((n \cdot \delta))] - \sin((b \cdot (n \cdot \gamma)) + \nu))$$

n =[1 . . n days]

d = distance to planet a.u.

b = ratio synodic period of planet to Earth year

nu = initial synodic longitude of epicycle in radians

delta = degrees of ecliptic longitude travel per day in radians

gamma = degrees of synodic longitude travel per day in epicycle in radians

Fig. 18 - Generic equation to generate an Apollonius-Ptolemaic "orbit" - Step 1

Ptolemy-Copernican Debate - Ptolemaic computer rendering - Earth-centered "orbit" of Mars

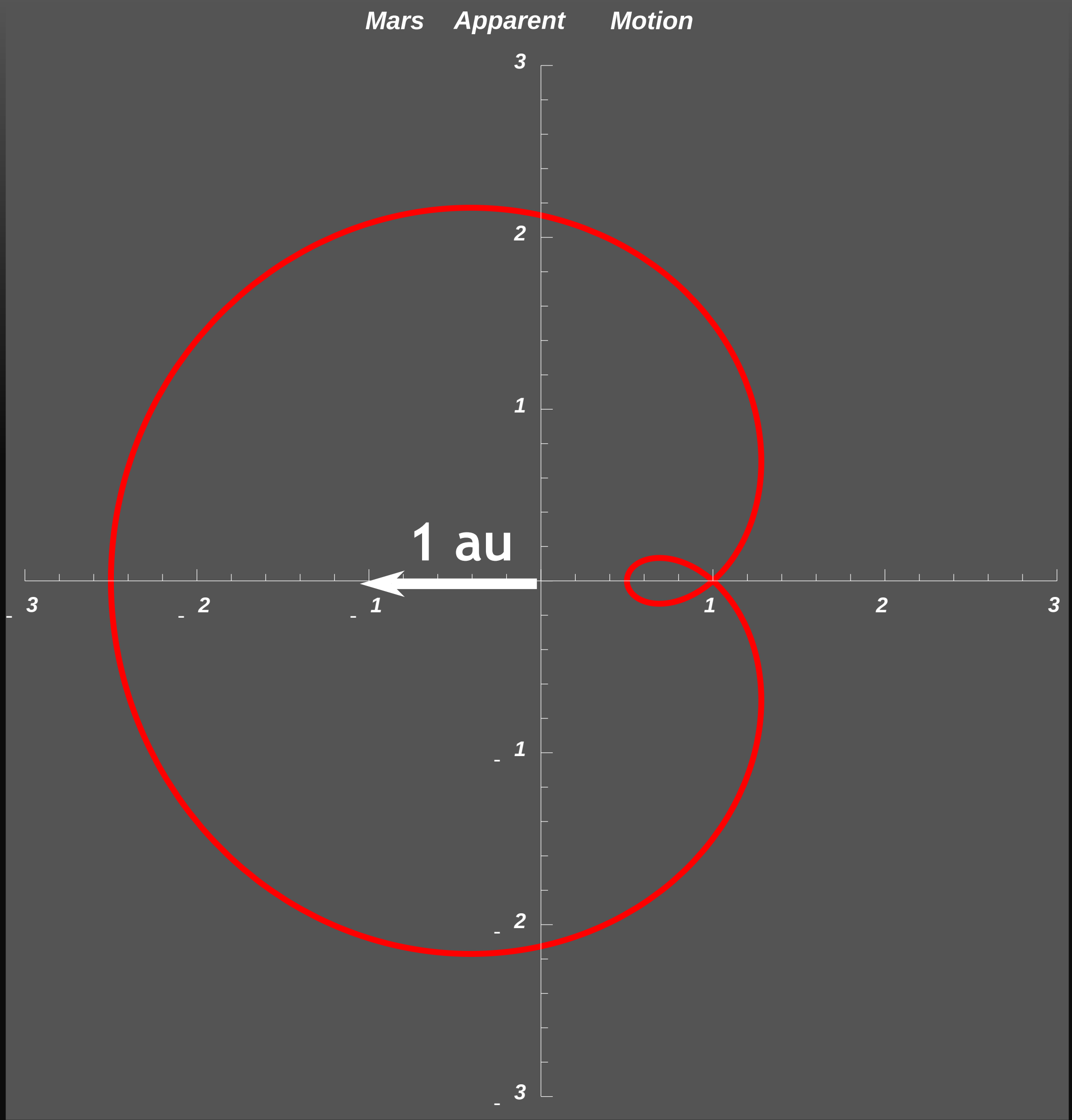


Fig. 19 - Orbit of Mars in the Ptolemaic system

Run animated version, Fig. 12 in browser.

Ptolemy-Copernican Debate - Ptolemaic model - figure rotation to ecliptic longitude of date

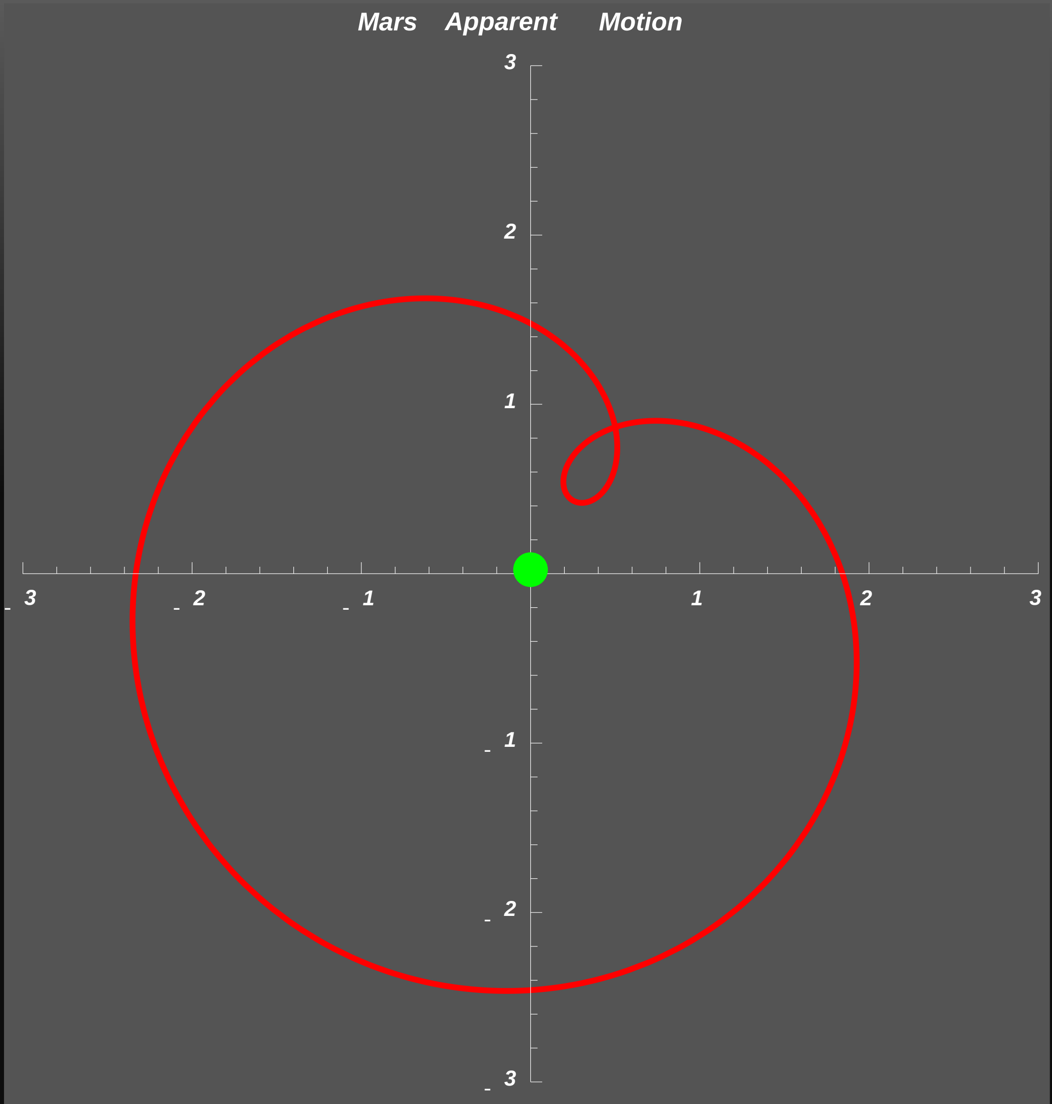


Fig. 20 - Orbits of Mars rotated to its eclipitic longitude

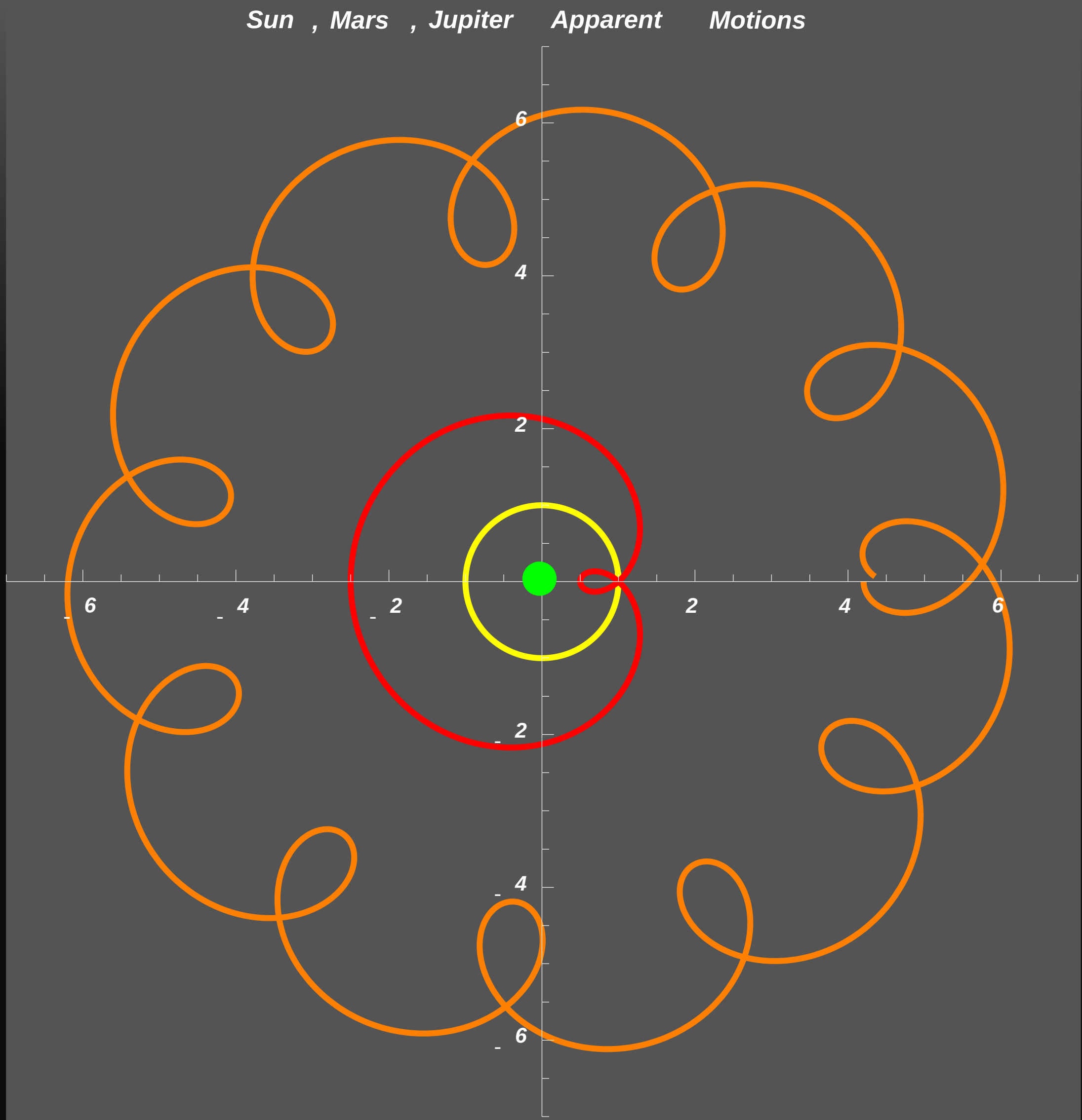
And then, in step 2, the orbit is rotated to the eclipitic longitude of date.

$$x' = [x \cdot \cos(-\lambda)] - [y \cdot \sin(-\lambda)]$$
$$y' = [-x \cdot \sin(-\lambda)] - [y \cdot \cos(-\lambda)]$$

Fig. 21 - Generic equation to generate an Apollonius-Ptolemaic "orbit" - Step 2 - coordinate rotation to ecliptic of date - lamba

Ptolemy-Copernican Debate - Ptolemaic model complexity - Orbits of the Sun, Mars and Jupiter

Now the Ptolemaic model starts to get complicated.



Run animated version, Fig. 22 in browser.

Fig. 22 - Orbits of the Sun, Mars and Jupiter in the Ptolemaic system

Ptolemy-Copernican Debate - Ptolemaic model - How accurate was the model?

The Ptolemaic model was reasonably accurate for naked-eye observing of Mercury, Venus, Jupiter and Saturn, but the model could not accurately predict the position of Mars to naked-eye observing tolerances, even with Ptolemy's improvement of adding the equant. Mars's orbit is too eccentric, too close to the Earth, and is too inclined to ecliptic plane. Saturn's orbit is just as eccentric and nearly as inclined as Mars, but it is further away, so the error in apparent position of Saturn was less. Ptolemy struggled with Mars's orbit, and he had only partial success in predicting it. Because the Earth-centered model did not include key components of real orbits - inclination to the ecliptic and elliptical shape - the Apollonius-Ptolemy model would never be able to accurately model Mars's apparent orbit as seen from Earth, even with Ptolemy's addition of the equant, or off-center planetary orbit, that was intended to simulate part of an elliptical orbit.

Ptolemy-Copernican Debate - Ptolemaic model - How accurate was the model?

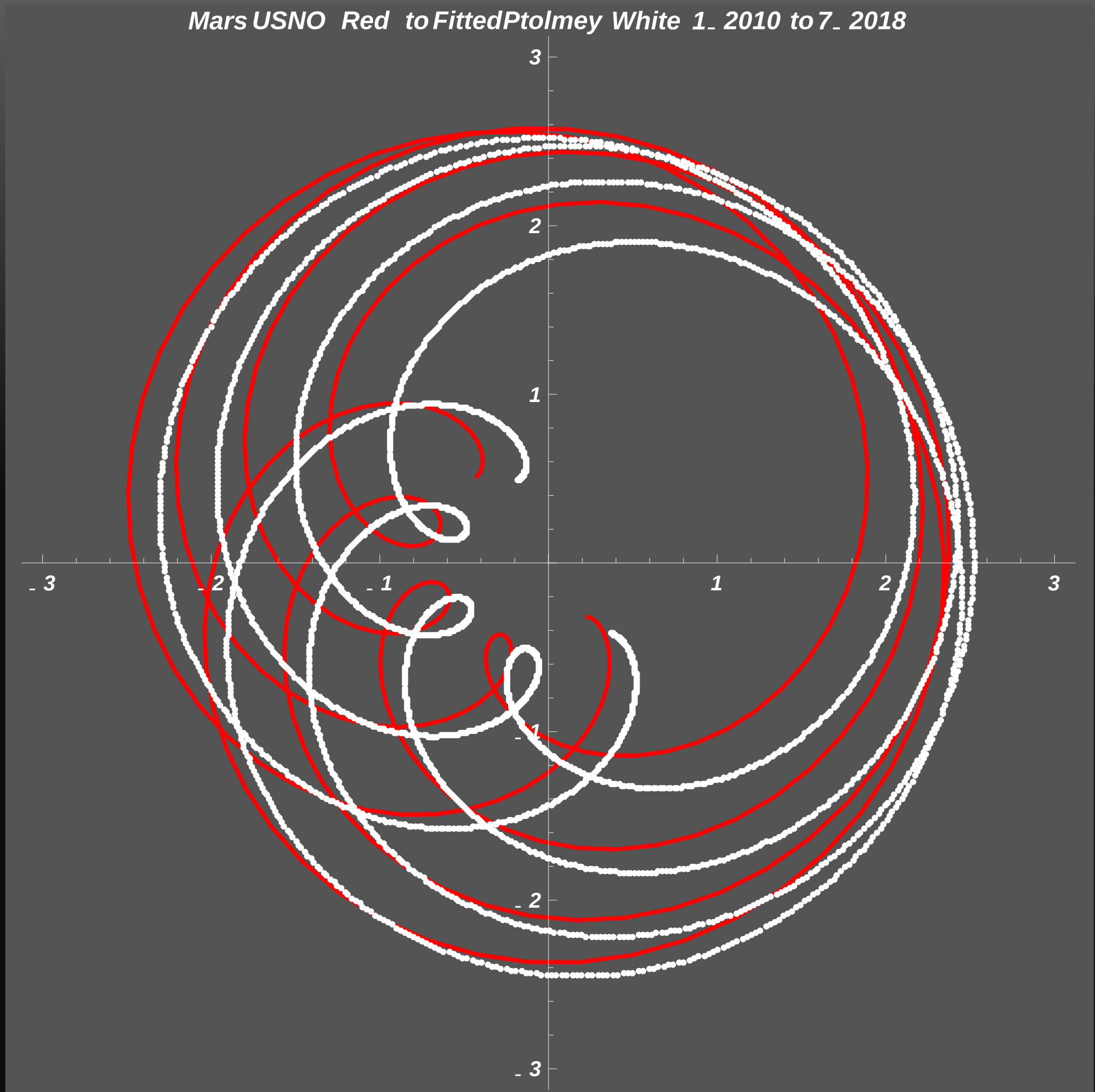


Fig. 23 - Comparison of predicted apparent motions for Mars 2010 to 2018. Source: UNSO MYAC Predicted; Author using prolate epitrochoid

The Apollonius simple prolate epitrochoid predicted orbit for Mars had errors in both the predicted date of the next opposition and the radius of the epicycle's retrograde motion.

Ptolemy by his invention of the equant, or off-centered planetary orbit, could either improve the perdicted periodicity of opposition or the radius of the epicycle's retrograde motion, but not both.

Ptolemy-Copernican Debate - Copernicus's reaction to the Ptolemaic model

The Apollonius-Ptolemaic models were:

1) Too complicated.
2) Used different methods to model uniform motions.
3) Were not sufficiently accurate.

Copernicus's closing jibe at the Ptolemaists:

"Mathematics is for the mathematicians[!]"

Copernicus in Preface, The Revolutions of the Heavens, qtd. in T. Kuhn, *The Copernican Revolution* (1957), p. 139.

Ptolemy-Copernican Debate - Copernicus's reaction to the Ptolemaic model: His apology to Pope Paul III (1543)

"That I allow the publication of these studies may surprise your Holiness . . . . How I came to dare to conceive such motion of the Earth, contrary to the received opinion of the Mathematicians [the Ptolemists] and indeed contrary to the impression of the senses, is what your Holiness will rather expect to hear. . . . . I was induced to think of a method of computing the motions of the spheres by nothing else than the knowledge that the Mathematicians are inconsistent in these investigations. For, first, the mathematicians are so unsure of the movements of the Sun and Moon that they cannot even explain or observe the constant length of the seasonal year. Secondly, they use neither the same principles and hypotheses nor the same demonstrations of the apparent motions . . . . So some use only homecentric circles, while others eccentrics and epicycles . . . . which seem to violate the first principle of uniformity of motion."

Preface, The Revolutions of the Heavens, qtd. in T. Kuhn, The Copernican Revolution (1957), pp. 138-139.

Ptolemy-Copernican Debate - Copernicus's purpose (1543)

"[But] I too began to think of the mobility of the Earth; and though the opinion seemed absurd, . . . I considered that I also might easily be allowed to try, by assuming some motion of the Earth, sounder explanations . . . ."

Preface, The Revolutions of the Heavens, qtd. in T. Kuhn, The Copernican Revolution (1957), pp. 142.

Copernican view - What if we put the Sun at center?

Keplerian parametric eq. for Mars and Earth in a

Sun-centered solar system

Mars

x = 1.5cos(t pi)
y = 1.5sin(t pi)

Earth

x = (1)cos(2t pi)

y = (1)sin(2t pi)

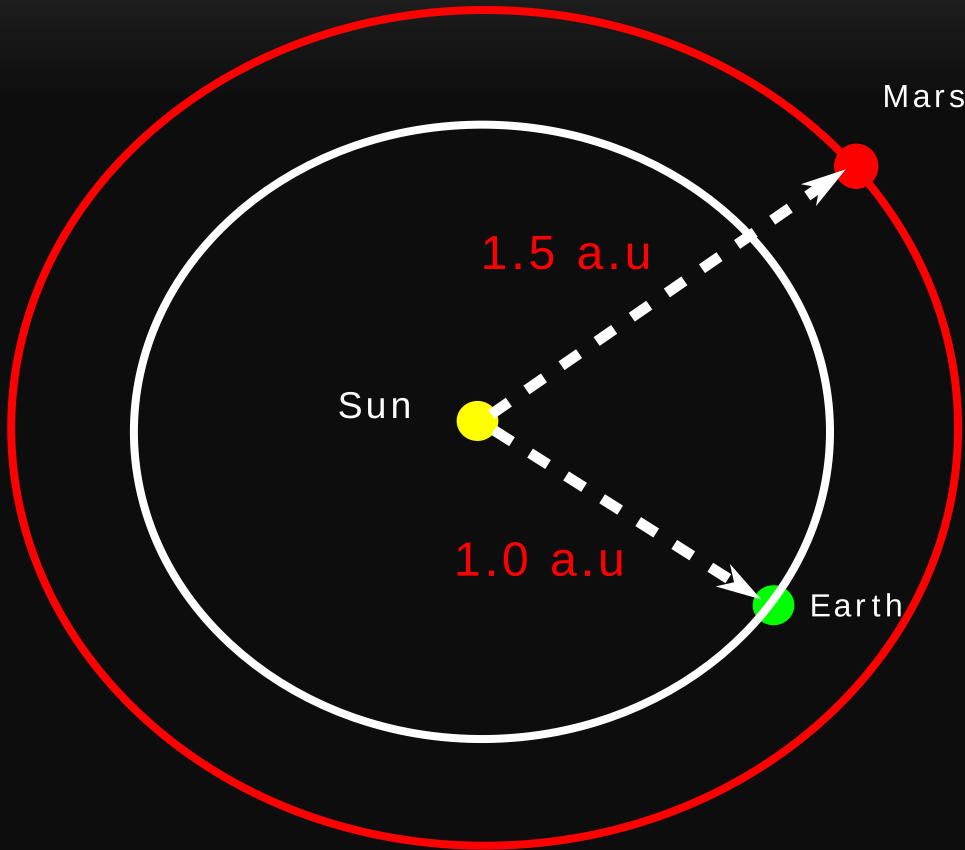


Fig. 25 - Copernican-Keplerian model

Ptolemy-Copernican Debate - A Computer rendering of the Copernican model

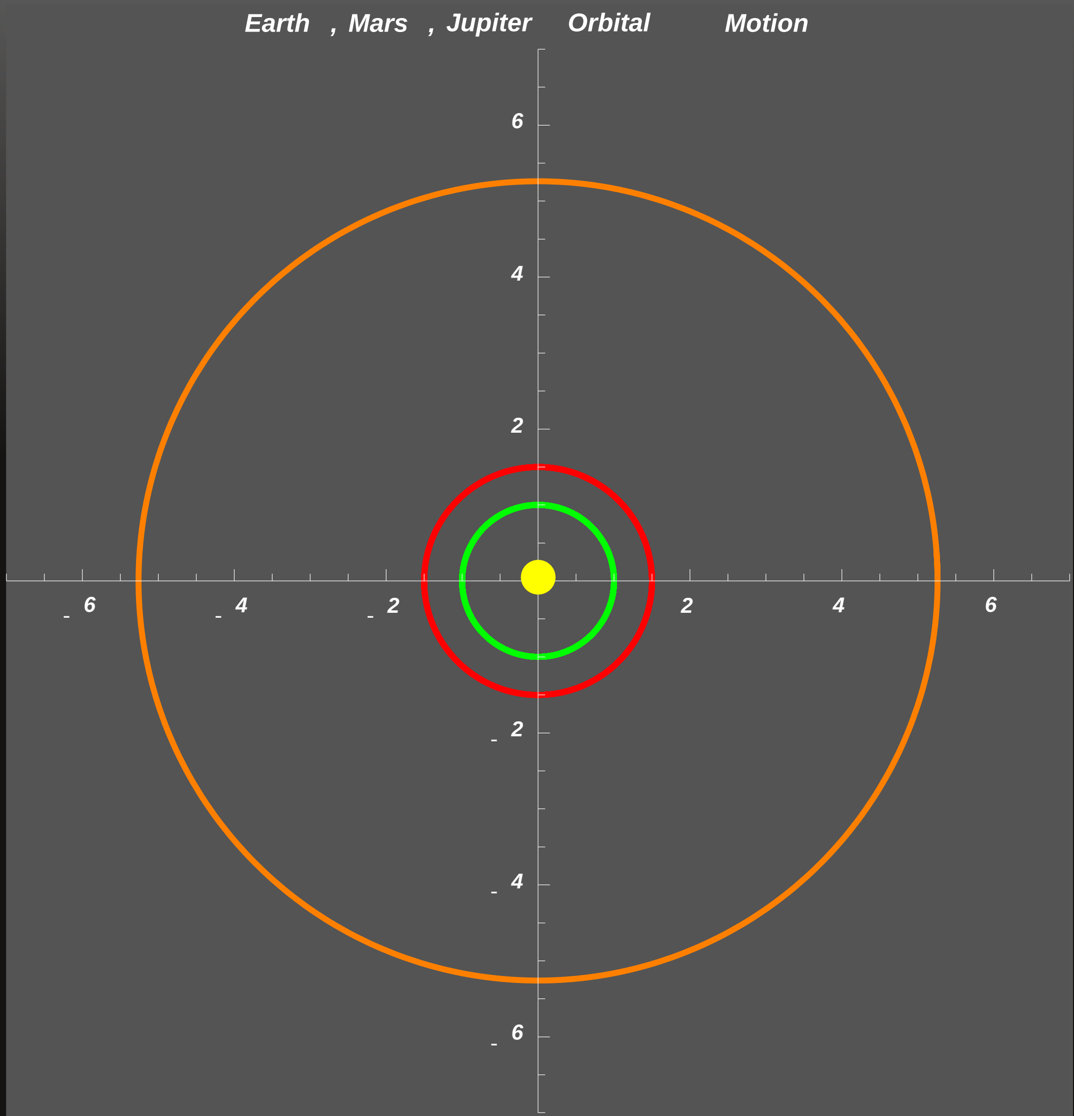


Fig. 26 - Copernican-Keplerian model

Run animated version, Fig. 26A in browser.

Ptolemy-Copernican Debate - Copernicus's problem

Copernicus needed to replicate the apparent motions of the planets as seen from Earth, including retrograde loops.

This is simple to prove using the mathematics of parametric equations -

Copernican-Keplerian model of the apparent motion of Mars from the Earth

The apparent motion of Mars as seen from Earth equals the position of Mars minus the position of the Earth.

Plus: Mars $x = 1.5\cos(t\,pi)$ $y = 1.5\sin(t\,pi)$

Minus: Earth $x = (1)\cos(2t\,pi)$ $y = (1)\sin(2t\,pi)$

$x = 1.5\cos(t\,pi)-(1)\cos(2t\,pi)$; $y = 1.5\sin(t\,pi)-(1)\sin(2t\,pi)$

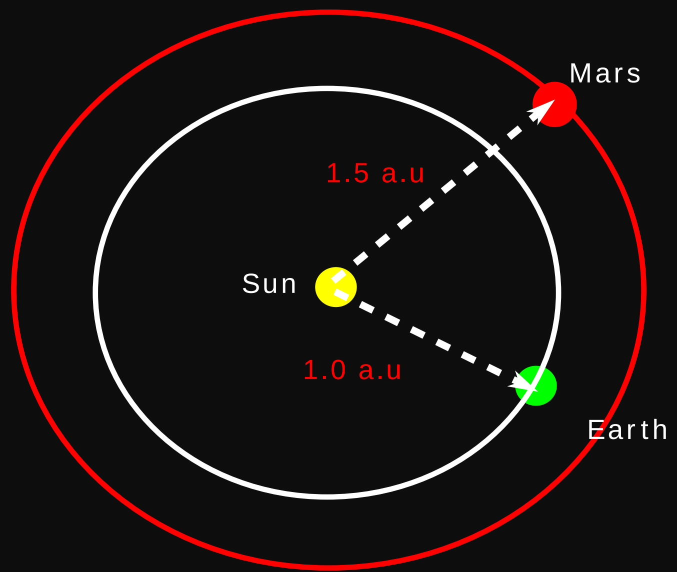


Fig. 25 - Copernican-Keplerian model

Ptolemy's Earth-centered model and Copernicus's Sun-centered plot the same apparent motion of Mars!

Ptolemy's equation for apparent motion of Mars for an Earth-centered solar system

x = 1.5cos(t pi) - (1)cos(2t pi)

y = 1.5sin(t pi) - (1)sin(2t pi)

Keplerian equation for apparent motion of Mars for a Sun-centered solar system

x = 1.5cos(t pi) - (1)cos(2t pi)

y = 1.5sin(t pi) - (1)sin(2t pi)

Ptolemy-Copernican Debate - Copernicus's "proof" of his model (1543)

After describing a Sun-centered universe and how that universe is more consistent with apparent motions of the planets, Copernicus retreats to Aristotleian reasoning: a model is true because it is the most "elegant" of available models:

"So we find underlying this ordination [the Sun-centered order of the planets] . . . a clear bond of harmony in the motion and magnitude of the Spheres such as can be discovered by no other wise."

Chapter 10, The Revolutions of the Heavens, qtd. in T. Ku, The Copernican Revolution (1957), pp. 180.

Ptolemy-Copernican Debate - Copernicus's "proof" of his model (1543)

Ptolemaic Complex, "Ugly" Model

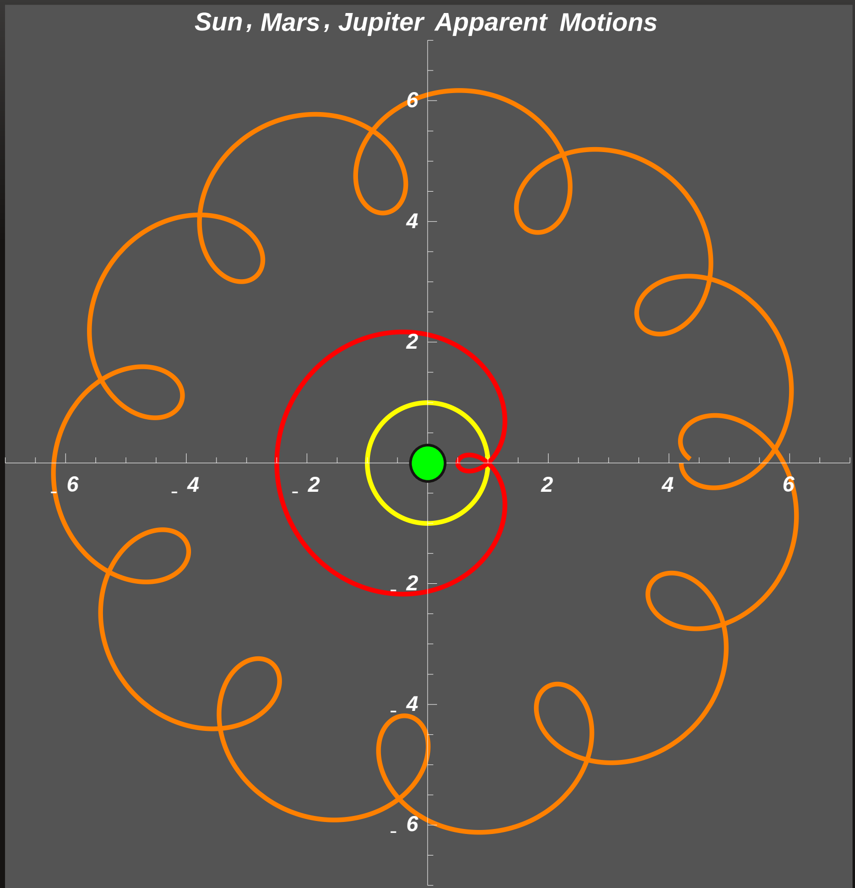


Fig. 27 - Orbits of Sun, Mars and Jupiter in the Ptolemaic model

Copernican Simple, "Beautiful" Model

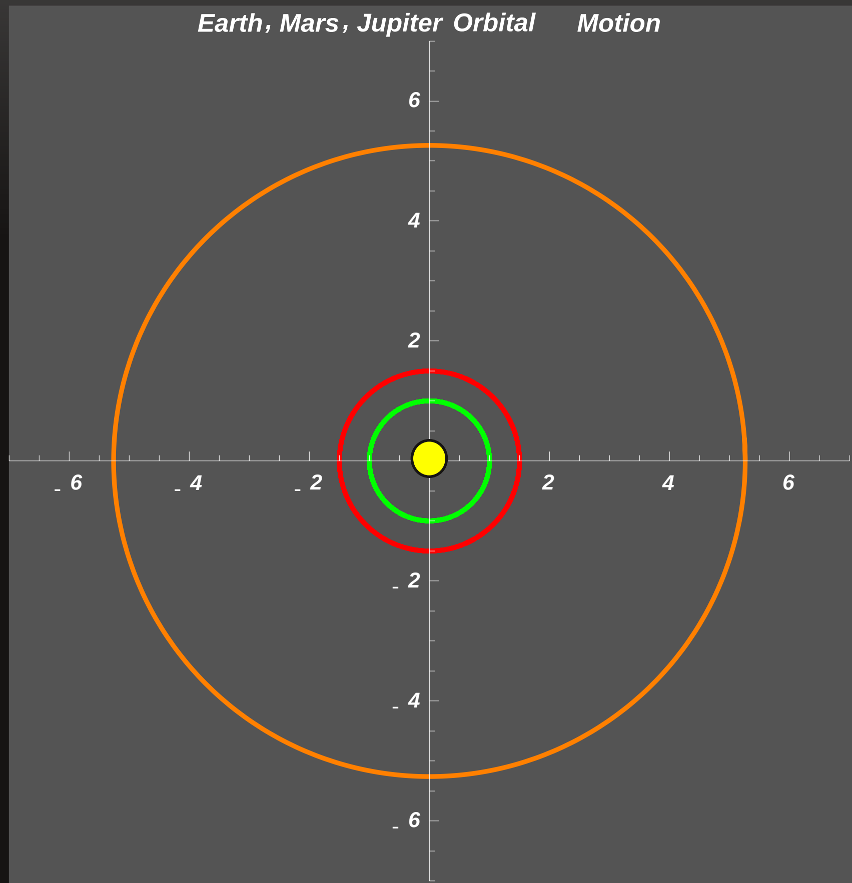


Fig. 28 - Copernican-Keplerian model

Run animated version of Fig.s 27a and 28a in browser.

Ptolemy-Copernican Debate - Venus's phases, A conclusive test of the Earth-centered solar system

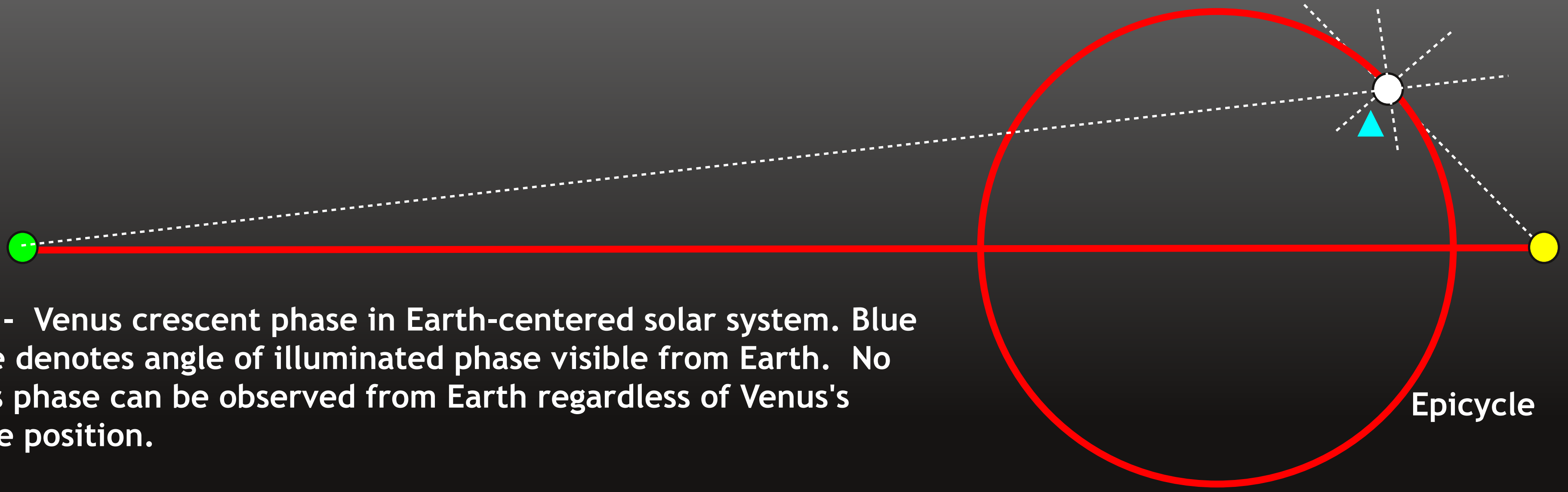


Fig. 29 - Venus crescent phase in Earth-centered solar system. Blue triangle denotes angle of illuminated phase visible from Earth. No gibbous phase can be observed from Earth regardless of Venus's epicycle position.

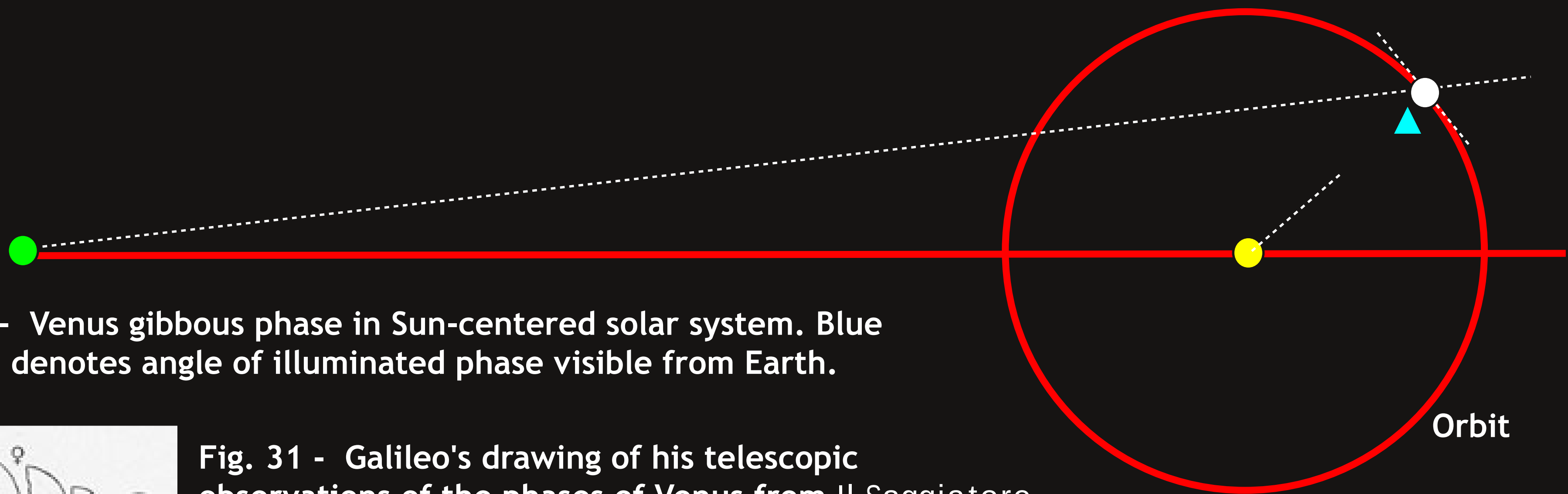


Fig. 30 - Venus gibbous phase in Sun-centered solar system. Blue triangle denotes angle of illuminated phase visible from Earth.

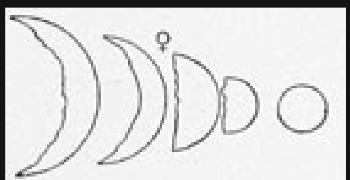


Fig. 31 - Galileo's drawing of his telescopic observations of the phases of Venus from Il Saggiatore (The Assayer) (1623)

Ptolemy-Copernican debate - Who won?

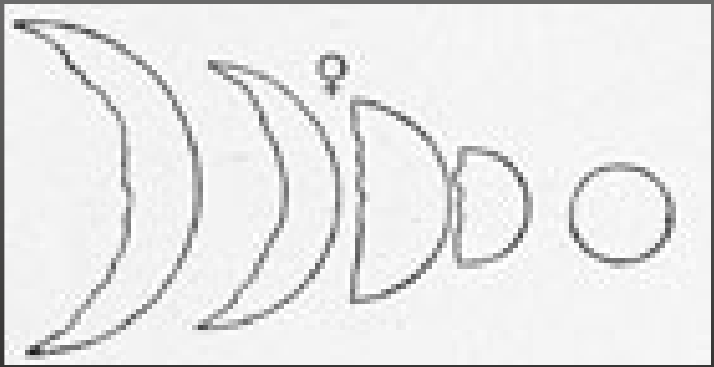


Fig. 31 - Galileo's drawing of his telescopic observations of the phases of Venus from Il Saggiatore **(The Assayer) (1623)**

Although the two models agreed for the motion of Mars, they differed for the motion of Venus. In a Ptolemaic Earth-centered solar system model, when viewed from Earth, Venus should only reach a crescent phase. In a Copernican Sun-centered solar system model, as viewed from Earth, Venus's phases go to gibbous illumination. In 1610, Galileo used a telescope to look at the phases of Venus, and he saw Venus proceed to a gibbous phase. But Galileo delays publication until 1623. Galileo's 1623 publication of his Venus observations conclusively ended the debate in favor of a Sun-centered solar system. In 1621, Kepler completed publication of heliocentric model of the orbit of Mars and planetary laws, thus, solving the key orbit problem that defeated Ptolemy. But many people continued to believe Ptolemaic model for the next 150 years because they were unwilling to accept that they were standing on the surface of a sphere that was rotating at 1,500 kph.

Ptolemy-Copernican Debate: Timeline - The Rise

262-190 B.C.E. - Apollonius of Perga formulates the deferent and epicycle theory of orbits.

130 B.C.E. - The Greek translation of the Hebrew Bible is completed, including the passage, 1 Chronicles 16:30: "He has fixed the earth firm, immovable."

150 C.E. - Followers of Ptolemy issue the first edition of Mathematical Treatise (The Almagest) and Planetary Hypotheses.

1517 - The distribution of non-latin bibles, made possible by Guttenberg's printing press, and Martin Luther's Ninety-Five Theses begin the era of individual conscience - the Reformation. As Lutheranism and Protestantism grows, religious wars break out across the Holy Roman Empire.

Ptolemy-Copernican Debate: Timeline - The Rise

1533 - Papal secretary Johann Widmanstadt writes Copernicus and encourages him to publish his theories on heliocentricism.

1536-1539 - Henry VIII seizes Catholic Church assets in England.

1543 - Deathbed publication of Copernicus's On the Revolution of the Spheres **that contains a preface addressed to the pope.**

Ptolemy-Copernican Debate: Timeline - The Fall

1545-1563 - The Holy Roman Empire begins the Counter-Reformation at the Council of Trent. Counter-Reformation doctrine requires strict interpretation of biblical text.

1555 - The Peace of Augsburg treaty legally divides the Holy Roman Empire into two parts: those European states in which the head-of-state will legally recognize Lutheranism and those who would only recognize Catholicism. Later, the Peace of Ausburg unravels and religious wars resume.

1600 - As part of Counter-Reformation supressions, the Roman Inquisition burns priest Giordano Bruno alive at the stake for promoting heliocentrisim.

Ptolemy-Copernican Debate: Timeline - The Challenge

1609 - Galileo makes his initial telescopic observations. Kepler publishes Astronomia Nova **including his improved heliocentric model for the orbit of Mars.**

1610 - Galileo publishes The Starry Messenger, **and he makes, but does not publish, his observations of the phases of Venus.**

1616, Feb. 24 - Consultant's Report to the Roman Inquisition concludes with respect to that the Sun is center of the world and that the Earth moves, "this proposition is foolish and absurd in philosophy, and formally heretical . . ."

1618-1648 - The 30 Years War rages across northern Europe resulting in German states losing 30% of their population.

Ptolemy-Copernican Debate: Timeline - The Challenge

1618-1621 - **Kepler publishes** The Epitome of Copernican Astronomy **containing his three laws of planetary motion.**

1623 - **Galileo publishes his 1610 Venus phase observations in** The Assayer.

1627 - **Kepler publishes** The Rudolphine Tables.

1632 - **Galileo publishes the** Dialogue Concerning the Two Chief World Systems.

Ptolemy-Copernican Debate: Timeline - The Suppression

1633, June 23 - Roman Inquisition convicts Galileo of heresy and sentences him to life house arrest.

1642 - Galileo passes after 8 years of house arrest.

1648 - The Peace of Westphalia treaty ends the Thirty Years War, and the treaty recognizes that each head-of-state may declare a state religion of Catholicism, Lutheranism or Calvinism. Any citizen of a state is free to regulated practice of their chosen faith (that is not the state declared religion) in public and an unlimited right to practice their chosen faith in private.

Ptolemy-Copernican Debate: Timeline - The Repentance

1992, Nov. 4 - Vatican publishes Galileo: Report on Papal Commission Findings, **by Cardinal Poupard, and Pope John Paul II concludes that,**

> **"Thanks to his intuition as a brilliant physicist and by relying on different arguments, Galileo, who practically invented the experimental method, understood why only the sun could function as the centre of the world, as it was then known, that is to say, as a planetary system. The error of the theologians of the time, when they maintained the centrality of the earth, was to think that our understanding of the physical world's structure as, in some way, imposed by the literal sense of Sacred Scripture."** L'Osservatore Romano **N. 44 (1264).**

2000 Easter, Day of Pardon - Following publication of a Vactican rexamination of the historical activities of Inquisition, Pope John Paul II formally apologizes to all victims of Inquisition and grants all persons historically convicted by the Inquisition a pardon, including but not specifically mentioning Galileo.

Lessons learned from the Ptolemy-Copernican debate for your future studies -

1. Mathematical models of reality are not reality. Do not confuse a mathematical model that you build with reality. Math modeling is useful to guide your intuition away from poorly formed notions about physical reality.

2. When building a model, try to include all signficant causal physical characteristics as parameters. Do not build partial models that arbitrarily exclude causal factors.

3. At times, more than two or more equally probable mathematical models can describe the world, but you have to be diligent in looking for the one that best describes real events. Always look for inconsistencies.

4. State or religion attempts to dictate the results of scientific investigations do not benefit the advance of humanity.

Ptolemy-Copernican debate - References

This presentation is an expanded and modified version of a video lecture given by Dr. Donald G. Saari, U.C. Irvine (2010) titled "Understanding Ptolemy's Enduring Achievement." Dr. Saari is the Chief Editor of the Bulletin of the American Mathematical Society. Saari, Donald G. (2010). The Power of Mathematical Thinking: From Newton's Laws to Election and the Economy. [Video]. Course No. 1417. The Teaching Company. Chantilly, Va.

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