Lambert Lunar Model and Sensitivity Testing of an Excel implementation of Carlotto's SfS algorithm

This short presentation provides background on:

1) Why the Lambert lunar model is not valid for solar altitudes above 30º; and,

2) How sensitive are SfS height measurements made using Carlotto's SfS algorithm to changes in the measured solar altitude?

In conclusion,

1) The formula used to model the Lambert lunar relationship is linear through 30º of solar altitude. Then the formula becomes non-linear and the relationship between slope and brightness breaksdown.

2) An one degree error in measurement of solar altitude translates into a 25% error in elevations computed in the resulting DEM.

Lambert lunar model

The Lambert lunar model is provided for in Carlotto's algorithm in the equation for converting pixel brightness to a slope:

$$Eg = (Pv - AvgPv) / (AvgPv * \tan(s))$$

The foregoing equation states that:

1) **a change in physical gradient =**

2) **a pixel's brightness less the average brightness of all pixels in a row, divided by,**

3) **the average brightness of all pixels in a row times the tangent of the solar zenith angle.**

This basic equation can be rearranged as:

Lambert lunar model

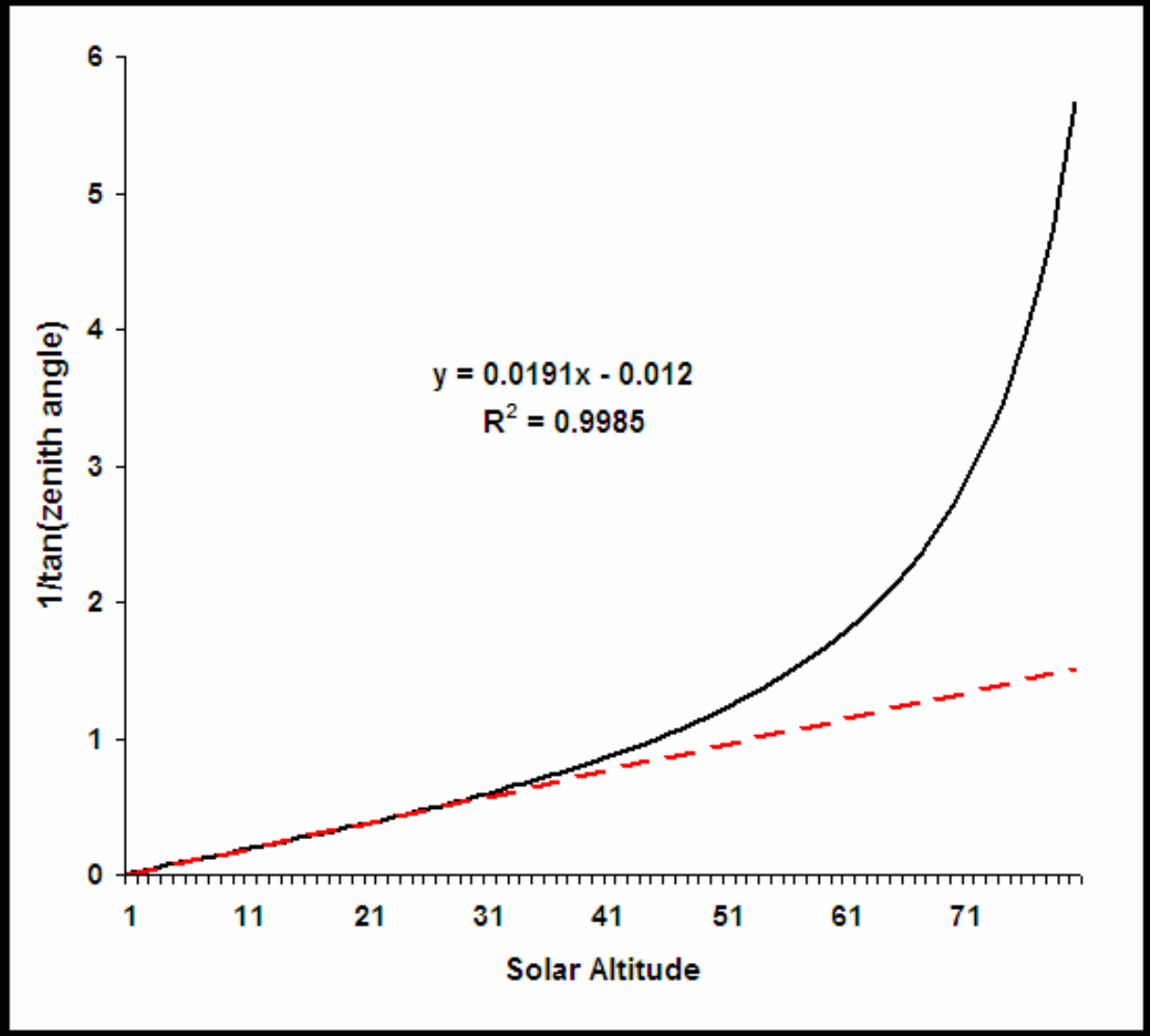
$$Eg = (Pv - AvgPv) * 1 / AvgPv / \tan(s)$$
$$= \text{slope} * \text{Lambert lunar factor}$$

The foregoing equation, the relationship between physical slope pixel brightness is modeled by:

$$1 / \tan(s)$$

This factor graphs as shown on the following slide. The graph of 1/ tan(zenith angle) is fitted with a trendline between solar altitudes 0º - 30º:

Lambert lunar model



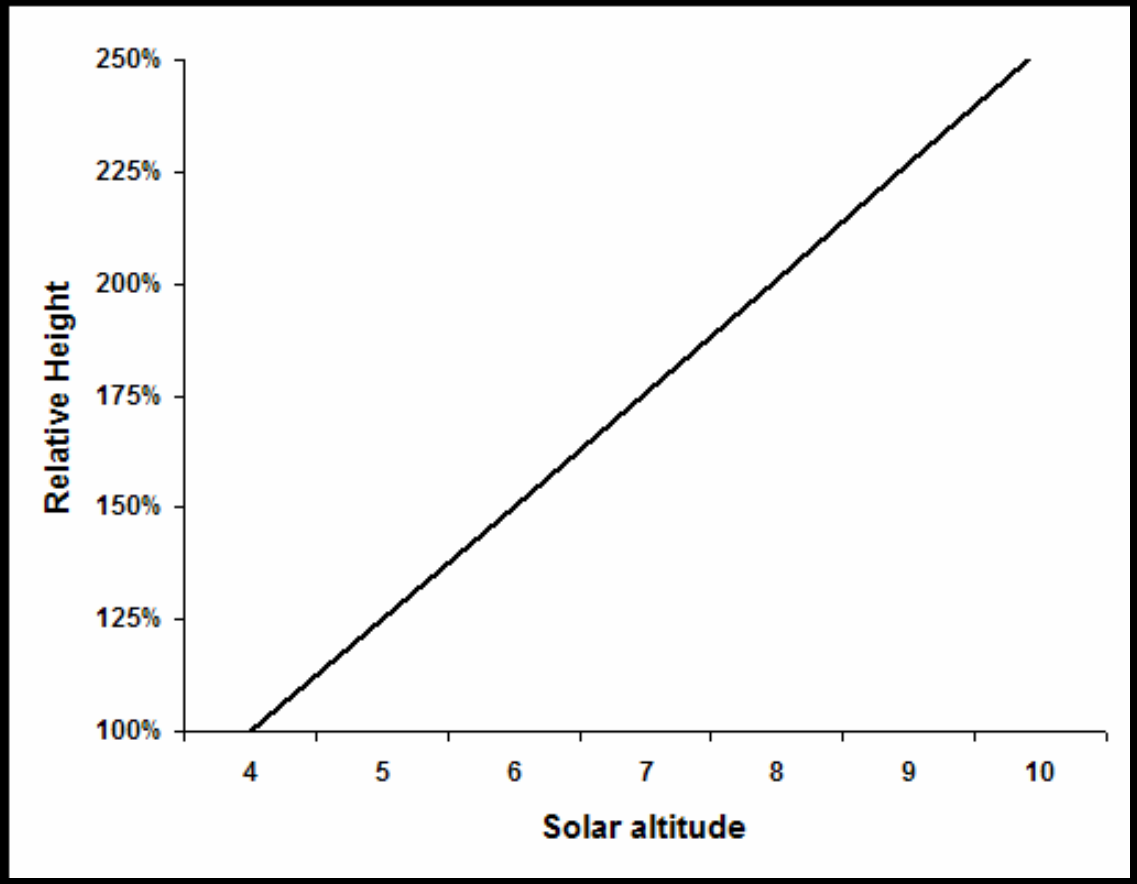
For altitudes 0º to 30º, there is good linear slope fit of 0.02 between the zenith angle or solar altitude and the Lambert lunar relationship. Beyond a solar altitude of 30º, the relationship breaks down.

Sensitivity testing

In the spreadsheet presented here – DEMCarlottoMethod.xls – is a synthetic ledger or rupes fault. Holding pixel brightness constant, the relative height of ledge computed using Carlotto's SfS algorithm increases 2 ½ times when the angle of the Sun is increased from 4º to 10º. 10º / 4º = 2 ½.

Sensitivity testing

This translates into the following rule-of-thumb. For each degree in error of measuring the Sun's altitude at the feature, final DEM elevations computed using Carlotto's method will be off by 25%:



The lesson from this sensitivity demonstration is that modelers should attempt to make the most accurate measurements of solar altitude that are possible and to make cross and multiple measurements.