



**Katima, Alaska**

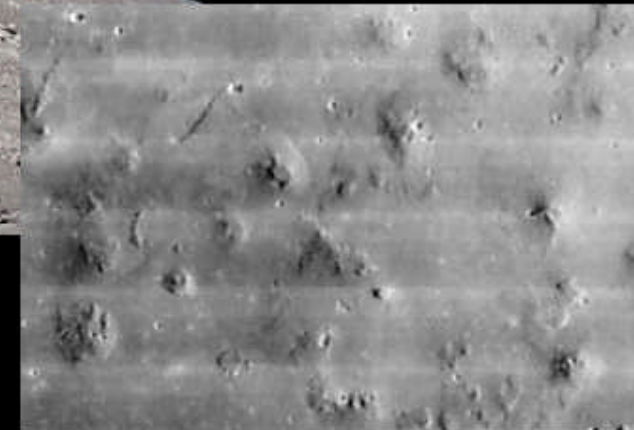
# **Simple Shape-from-shading (SfS) with Carlotto's method and MS-Excel**

## **Part 1: SfS concepts and terms**

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**Mons Rumker - Apollo 15 AS15-97-13252**



**Marius Hills - Lunar Orbiter IV - IV-15-H2**

**Advanced SfS users - skip ahead**

**Advanced Shape-from-Shading users can skip this presentation by:**

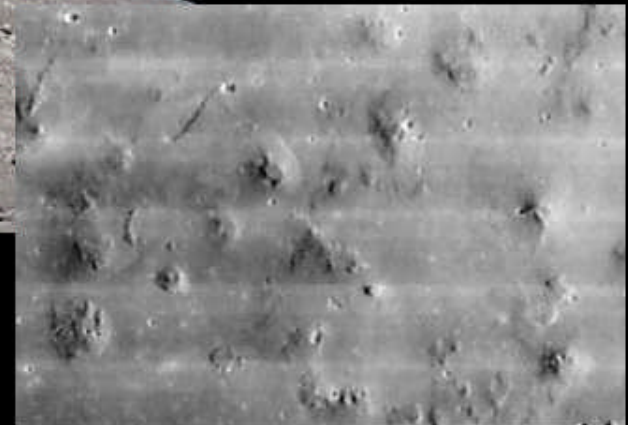
- 1) Review the "Learning Objectives" slide - two slides forward from this point.**
- 2) Review the last four slides of this presentation, which review the key concepts to be learned.**
- 3) Proceed to Part 2.**



**Katima, Alaska**



**Mons Rumker - Apollo 15 AS15-97-13252**

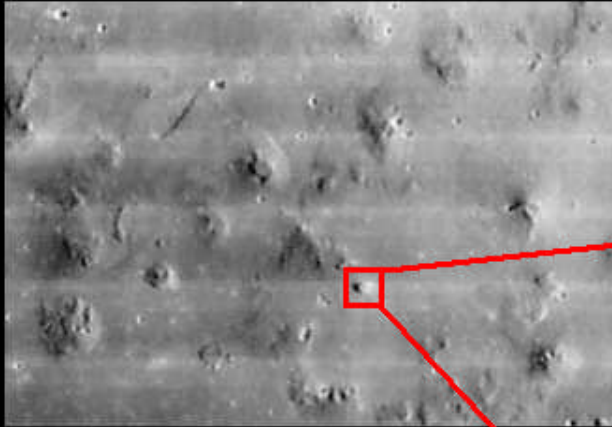


**Marius Hills - Lunar Orbiter IV - IV-15-H2**

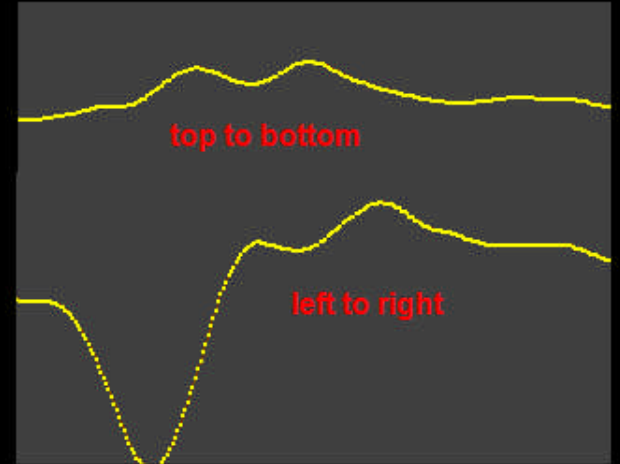
## **Learning objectives for this part:**

- 1) What basic nomenclature and terminology is used when discussing Shape-from-Shading (SfS)?**
- 2) How do simple SfS algorithms convert brightness to gradients and gradients to elevations in meters?**
- 3) What SfS assumptions restrict the types of lunar features that it can be applied to?**

**Image**



**Cropped image pixel map**



**Light profile curve**

**3-D Digital elevation model**



## Cropped image pixel map



## Convert image pixel values to 3-D x,y,z points

Microsoft Excel - DFWCarlottoInMedEd.xls

File Edit View Insert Format Tools Data Window Help PDF Creator

Shape from Shading Utility after Carlotto 1996 and Evans 2005 Version: pre-alpha test 0.2

in Microsoft Office in Windows (not compatible with MS Office in Mac)

A Process image to digital elevation map

Step 1 - Paste source image's luminance pixel values into worksheet ImagePixelValueMap as an NxM matrix

Step 2 - Identify the range of the image pixel values to this spreadsheet

2a Enter the upper-left-hand pixel cell name:  or absolute cell reference, e.g. B4D2

2b Enter the lower-right-hand pixel cell name:  or absolute cell reference, e.g. B7E1D0

2c Click button to identify and name the range:

Name pixel range:  Current value:

Clear Image Pixel Map

Step 3 - Enter image characteristics

3a Image scale:  meters/meter  meters/pixel uncertainty

3b Focal altitude:  degrees  7min angle degrees

3c Image topan radius:  meters or kilometers

3d Refreshing tilt adjustment to height:  Enter 0 if image is rectified, no refreshing

3e Difference between feature line and ground (L) less observer's lunar d:  degrees

3f Difference between feature lunar altitude (L) less observer's lunar d:  degrees

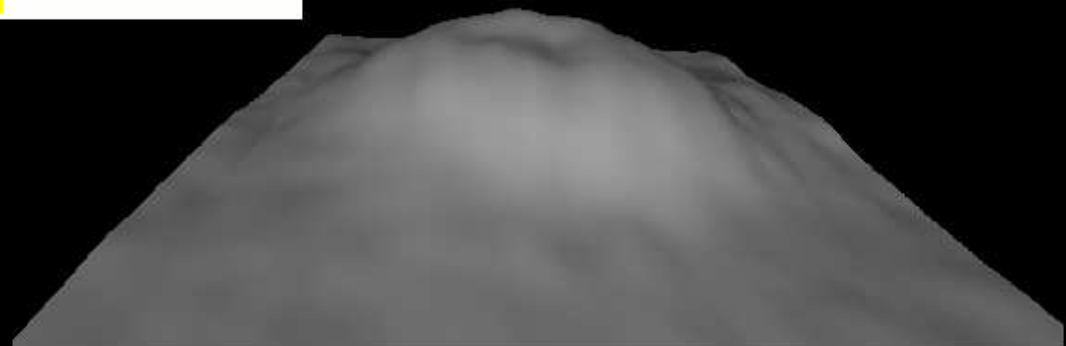
Step 4 - Generate the digital elevation map

4a Build DLM with procedure:  clicks

4b Click button to generate DFM

Generate DFM Process status:  1

## 3-D Digital elevation model



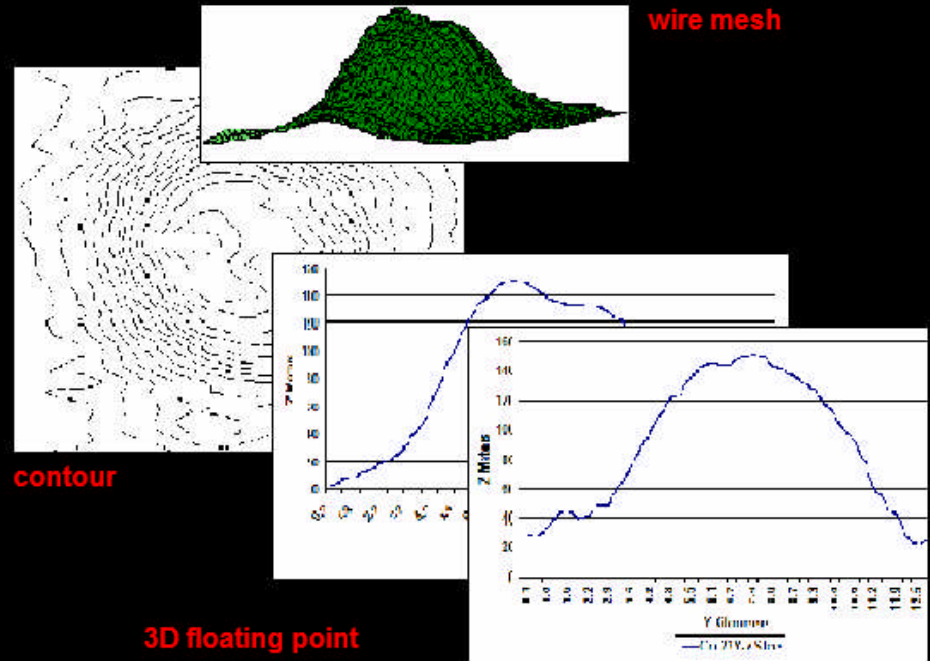
**Cropped image pixel map**



- is converted into a DEM -

41				
42	Step 5 - Optionally generate output formats			
43				
44		Make DEM Point Text Files		This utility
45				
46				
47	For external application	Text Type		
48	ETter	Point stream		
49				
50	Various	Trips: pixels x-y		
52		Z - real CSV		
53				
54	VARIOUS	Trips: CSV		
55		x,y,z real		
56				
57	VRML	Elevation grid		
58		x-y pixels, z-real		
59				
60				
61	VRML	3-D points		
62		x,y,z real		
63				
64				

- that is turned into many graphical representations using 3rd party applications chosen by the user.



**wire mesh**

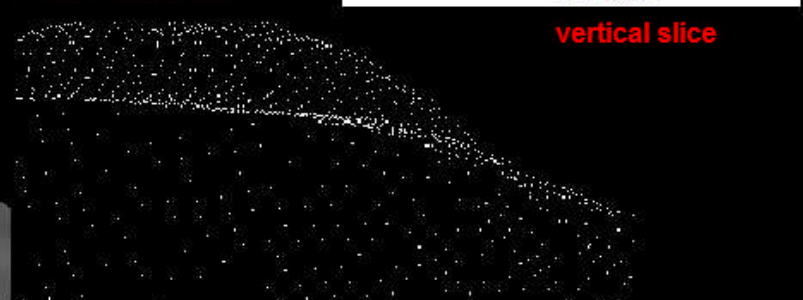
**contour**

**vertical slice**

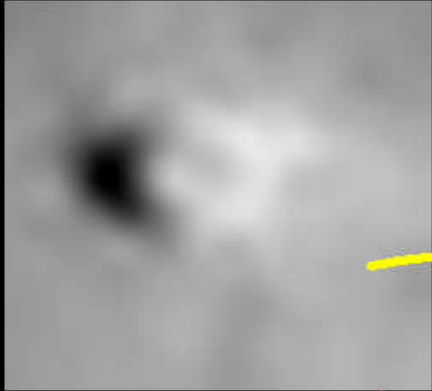
**3D elevation grid**



**3D floating point**



Cropped image pixel map



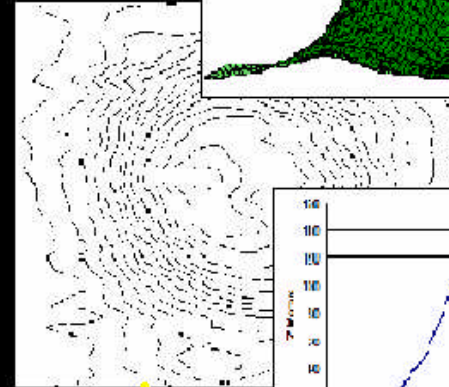
- is converted into a DEM - **EXCEL**

42	Step 5 - Optionally generate output formats	
43		
44	Make DEM Point Text Files	This utility
45		
46		
47	For external application	Text Type
48	Euler	Point stream
49		
50	Various	Trips: pixels x-y Z - real CSV
51		
52	Various	Trips: CSV x,y,z real
53		
54	VRML	Elevation grid x-y pixels, z-real
55		
56		
57	VRML	Elevation grid x-y pixels, z-real
58		
59		
60		
61	VRML	3-D points x,y,z real
62		
63		
64		

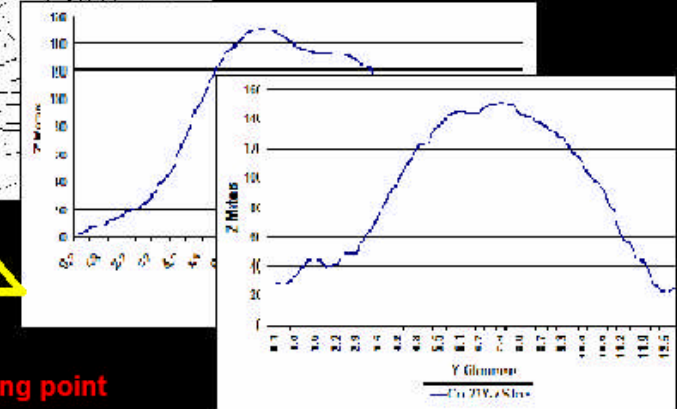
- that is turned into many graphical representations using 3rd party applications chosen by the user.



wire mesh  
**EULER**



contour



vertical slice  
**EXCEL**

JPG->FITS  
LTVT  
AIP4WIN

TEXT

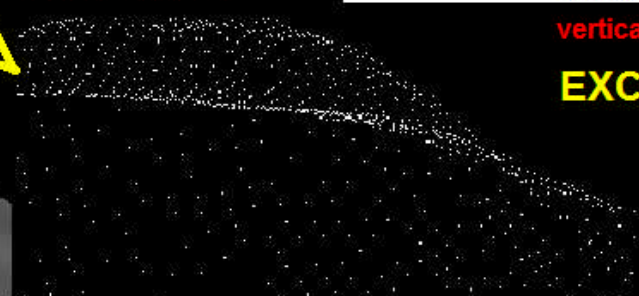


VRML

3D elevation grid



3D floating point





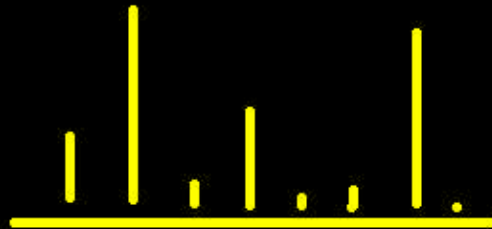
The original picture -



image pixel map

← 30 kilometers and 100 pixels →

- is digitized into brightness values



reflectance or incidence map (brightness per pixel)

- that are assumed to represent the slope of the terrain.



gradient map or map of slopes ( z pixel / 1 pixel)

Since the slopes are physically next to each other, they are linked, or summed, into a unitary gradient map



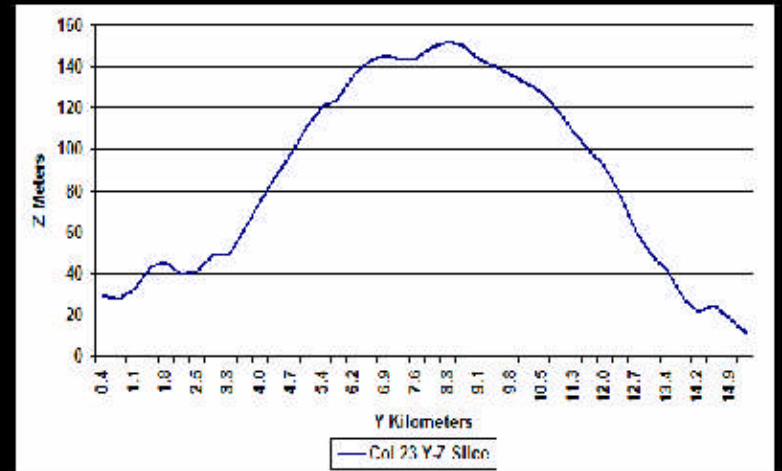
summed gradient map ( z pixel / 1 pixel)

The horizontal image scale is applied to the unitary gradient map, turning the unitary gradient map -

Horizontal image scale (30,000 meters / 100 pixels = 300 meters / 1 pixel)



summed gradient map ( z pixel / 1 pixel)



- into a digital elevation model or DEM. ( h meters by x kilometers)

**Complex**

**vs.**

**Simplified**

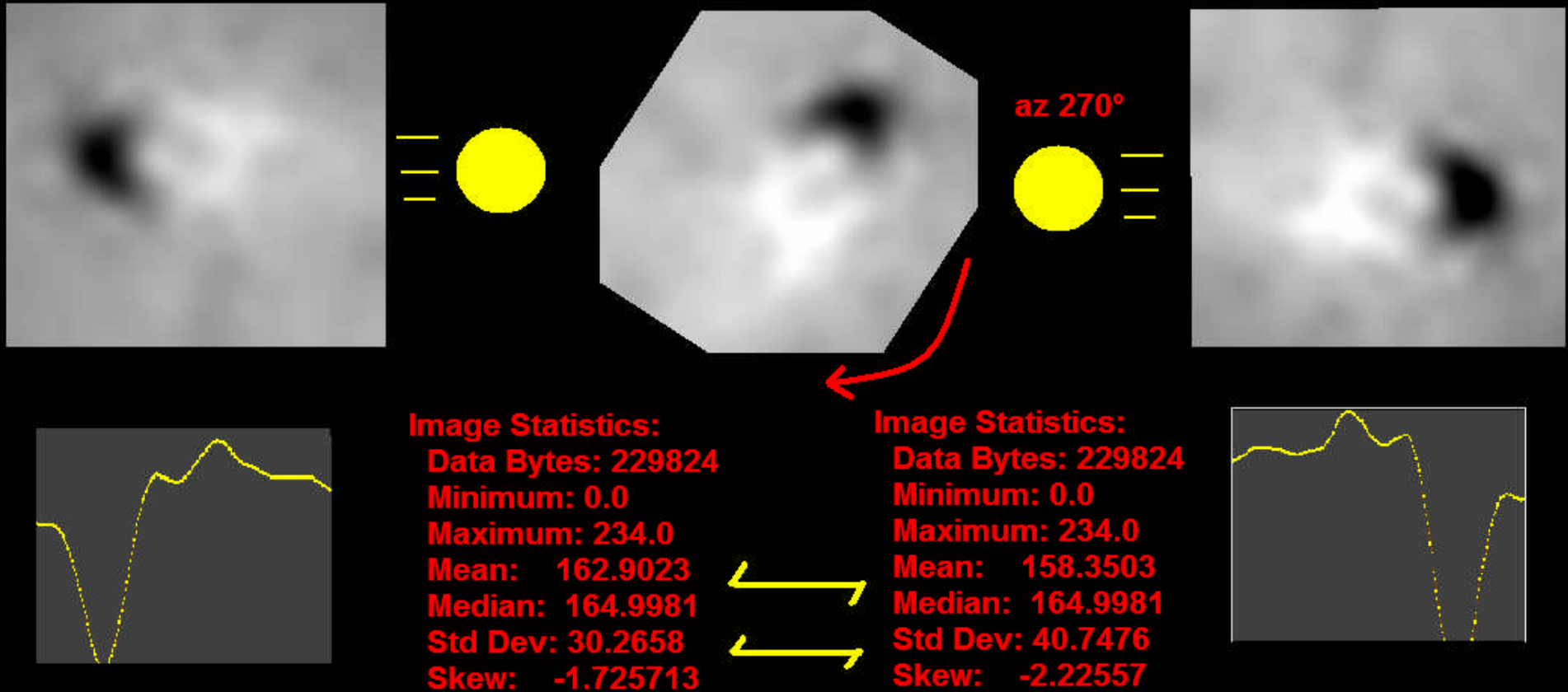
$$l(u,v) = p(u,v) * \{ (1+p_s*p(u,v)+ q_s*q(u,v) / [\text{Sqrt}(1 + p_s^2 + q_s^2)] * [\text{Sqrt}(1 + p^2(u,v) + q^2(u,v)) ] \}$$

$$Eg=(Pv - AvgPv)/(AvgPv*\tan(s))$$

**algorithms trade accuracy and flexibility against ease of implementation and restricted use cases.**

Carlotto's 1996 algorithm simplifies computation by rotating all images so the apparent azimuth of the Sun is at  $270^\circ$  - or flowing from the left to the right.

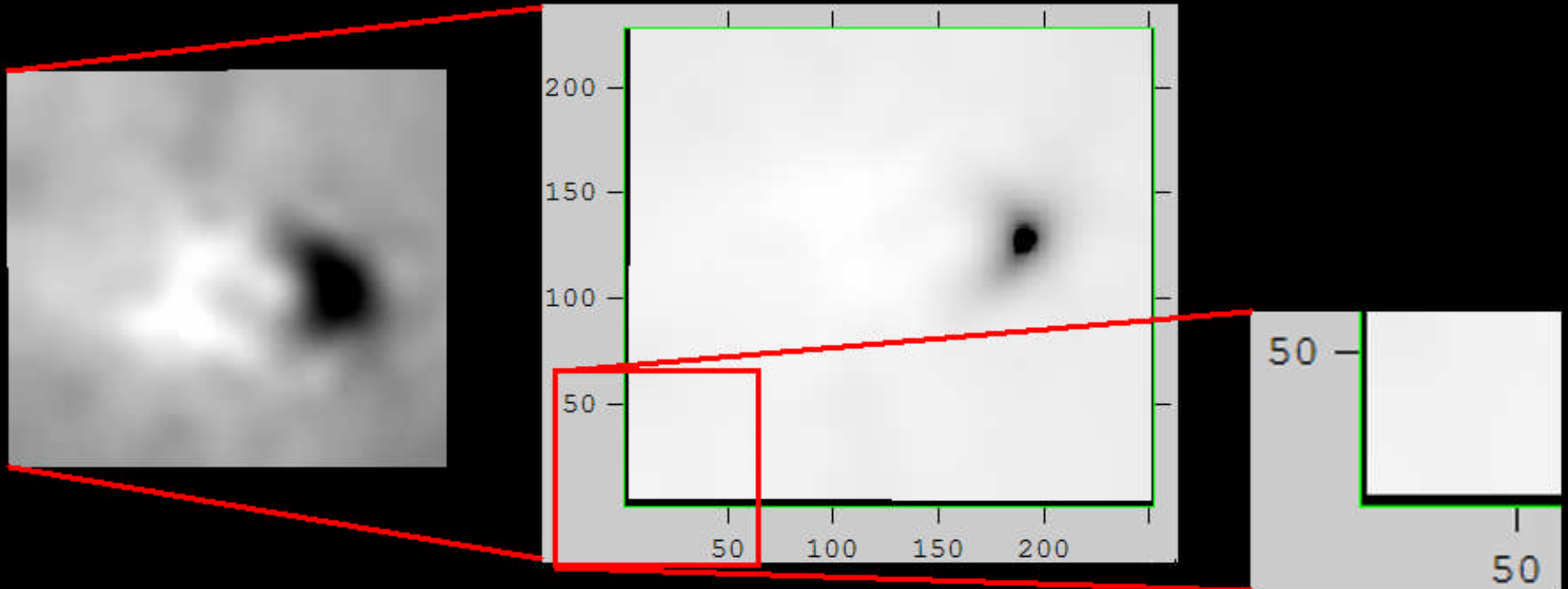
fits -> Astronomical Image Processing for Windows (AIP4WIN) -> fits



Rotation raises the concern that image processing modifies the image's photometry.

## Rotation artifacts

In this example, the source of the image change is easily identified as a row and column of black pixels inserted by rotation image processing. These artifacts can be easily removed by clipping, either before or after processing with the SfS spreadsheet.



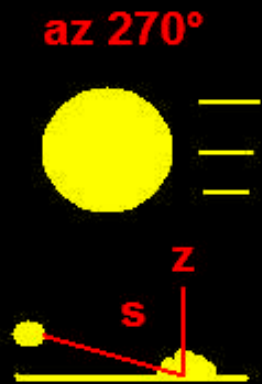
raw fits - rotated AIP4WIN

FV Tools

FV Tools

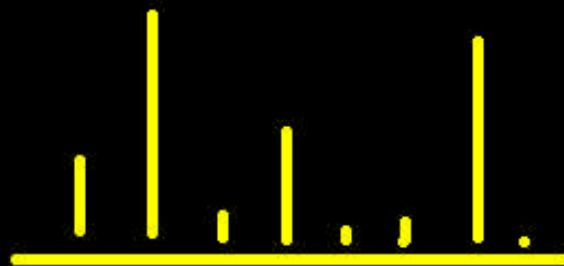
However, image rotation normally results in blank pixels in the rotated image. Image processing software, like AIP4WIN, fills these pixels by average interpolation from surrounding pixels.

Ideally, image rotation should be minimized, or confined to simple horizontal or vertical flips.



Once the feature image is rotated to the assumed apparent solar azimuth of 270°, conversion of pixel values to gradients (slopes) can be done with a simple slope equation.

$$(P_v - P_{vRowAvg}) / (P_{vRowAvg} * \tan(s)) = E_g \quad : \text{Elevation gradient}$$



reflectance or incidence map  
(brightness per pixel)



gradient map or map of slopes  
( z pixel / 1 pixel)

$$(y_2 - y_1) / (x_2 - x_1)$$

$$= \text{Slope}$$

## **Constraints when using simple SfS and Carlotto's algorithm**

**Knowing constraining assumptions used in SfS will allow you to select features from which you can create an accurate digital elevation model.**

- 1) Uniform surface reflectance**
- 2) Uniform low-angle illumination**
- 3) Rectangular FOV**
- 4) Equal spacing of x-y plane pixels (rectification)**
- 5) Foreshortening**
- 6) Image pixel dimensions and processing time**

## Constraint - uniform surface reflectance

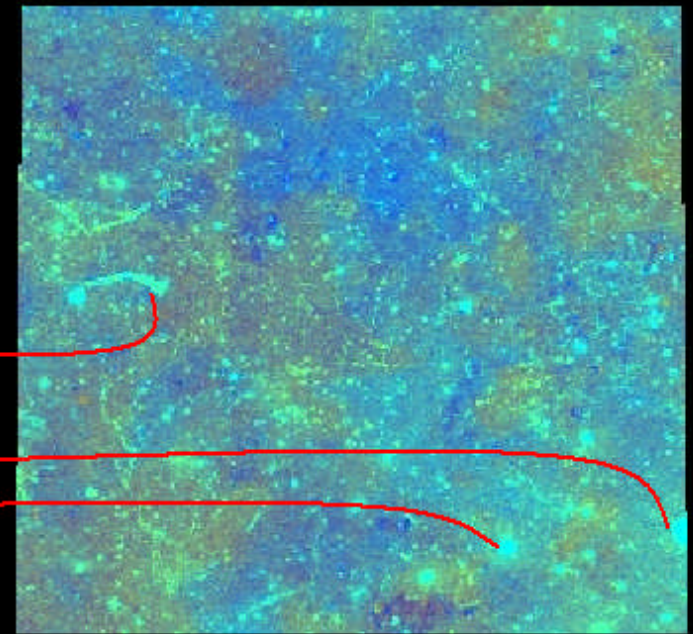
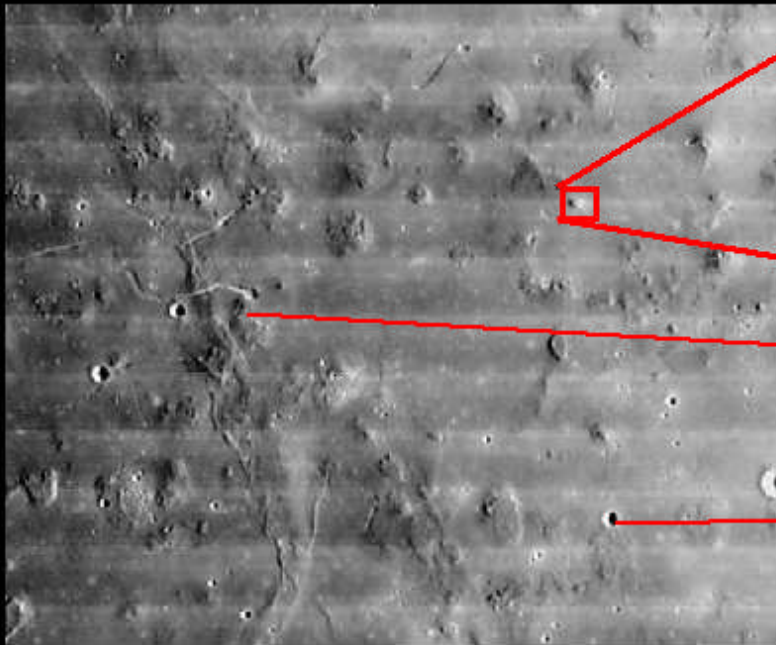
A reduced level of computation is achieved, in part, by assuming that the lunar surface to be measured is a uniform diffuse-lightscattering Lambertian surface.



Many lunar features will not meet this basic criteria because they have albedo variations related to the changes in surface mineral composition.

Does the reflectance change in this Lunar Orbiter IV image of the Marius Hills solely because of slope -

- or also because of changes in surface mineral composition, as seen in this Clementine false color ratio image of the same region?

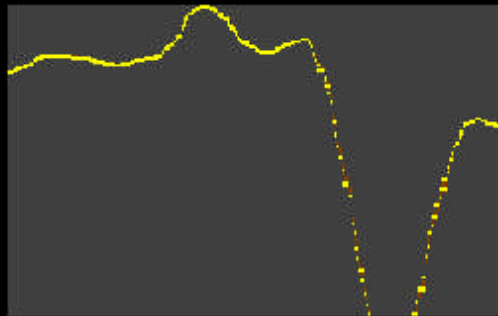


## Constraint - uniform low-angle illumination

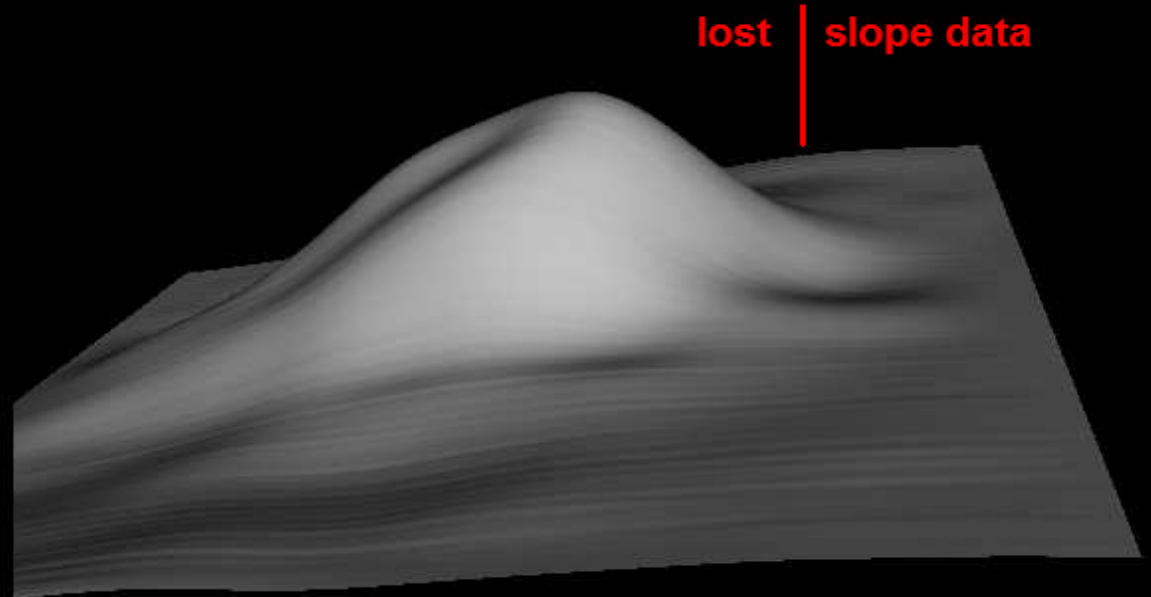
The feature needs to be bathed in low-angle illumination that returns grey tones within the pixel range of the image's format.

Images that have dark deep shadows may drop-out slope information.

The resulting feature DEM has a distorted appearance that does not accurately represent terrain changes.



lost slope data



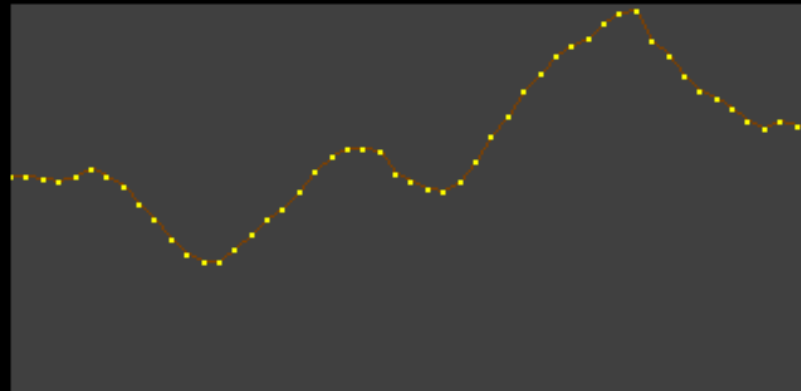
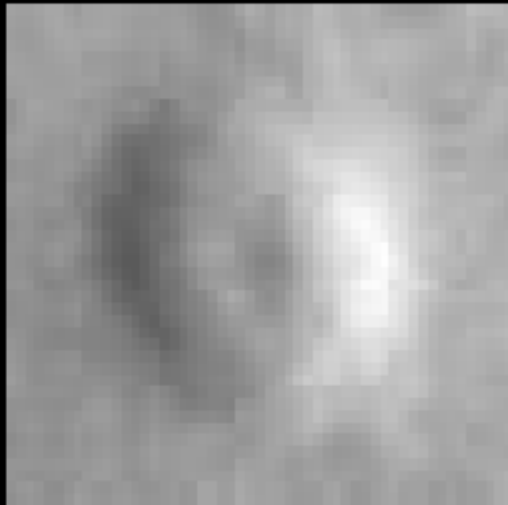
elevation grid - z-plane 3x



## Constraints - uniform low-angle illumination

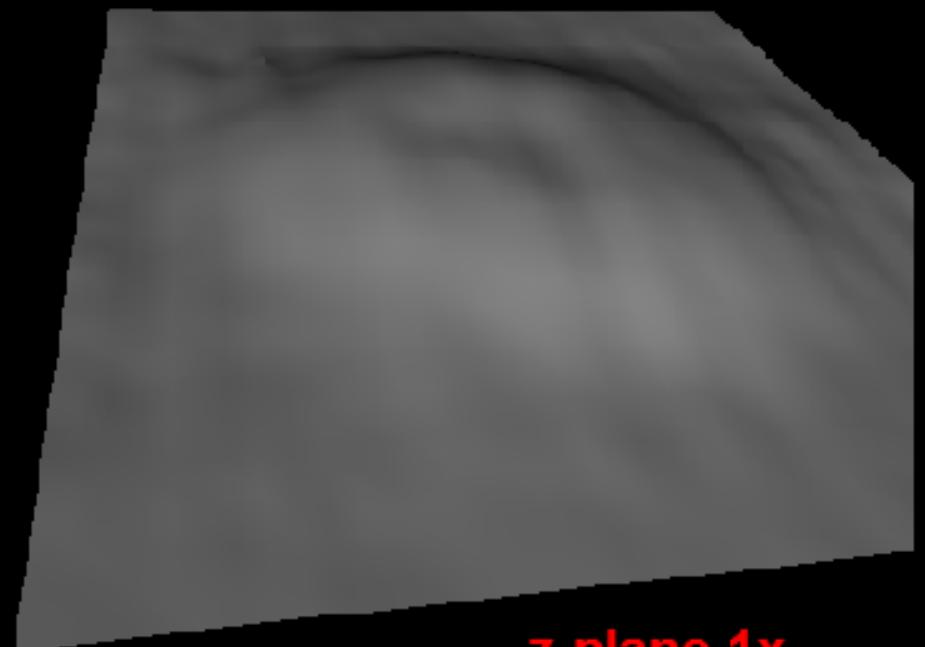
Features that have uniform low-angle illumination without shadow dropouts or bright spots yield the most accurate feature DEMs.

4° to 10° of solar altitude appears to work best.



Cauchy Omega

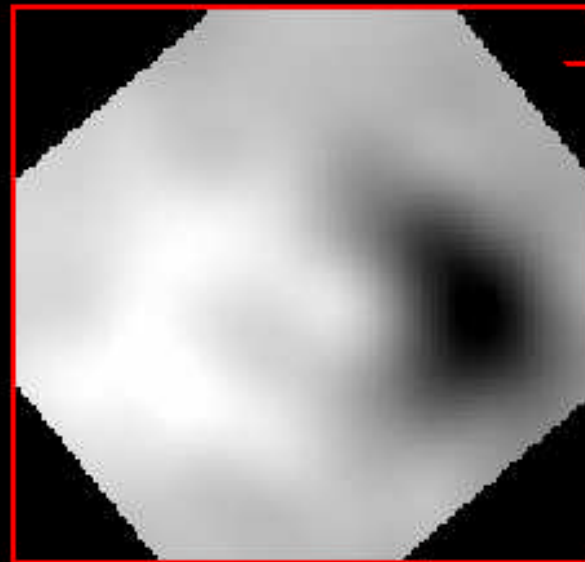
Image by Paolo  
Lazarotti from  
Evans 2006b



z-plane 1x

## Rectangular Field-of-view (FOV)

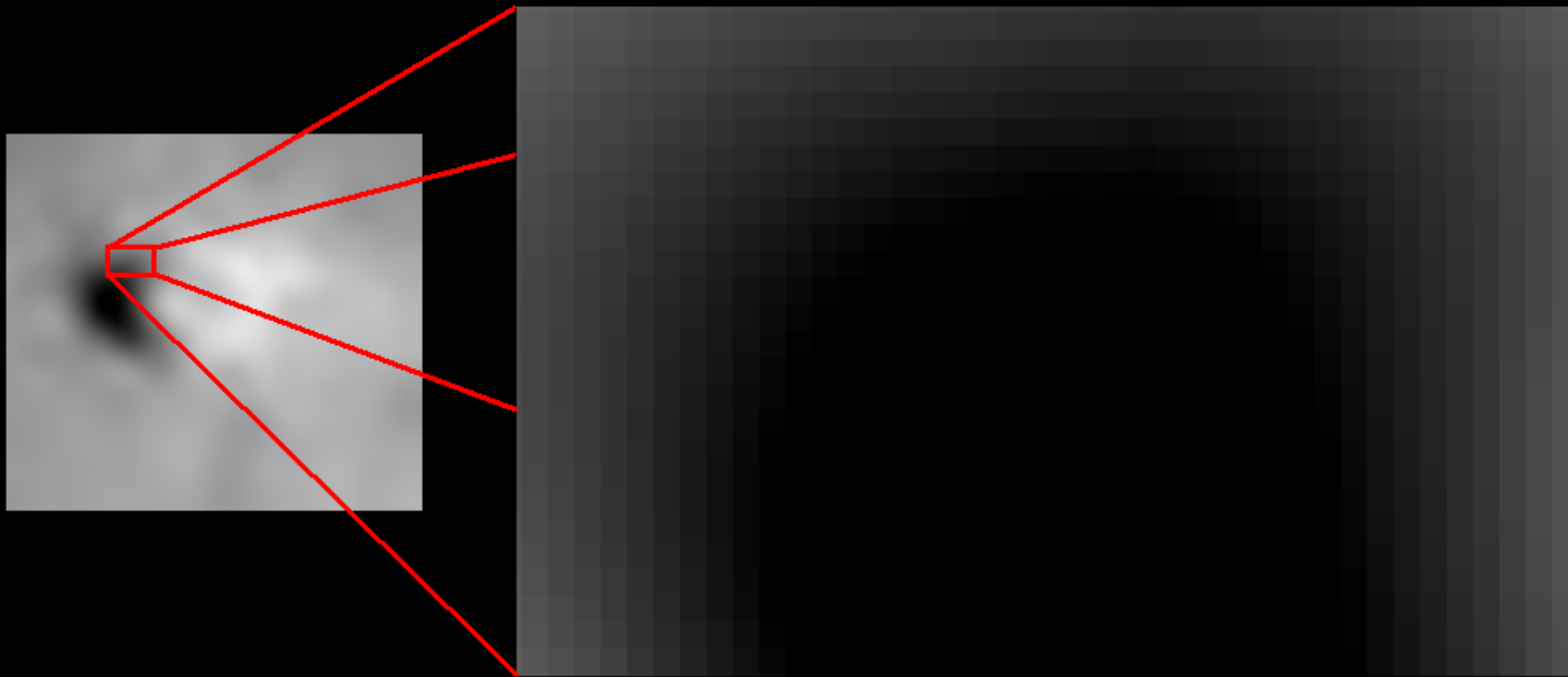
As noted in prior slides, Carlotto's method assumes the image has been rotated so the apparent solar angle is  $270^\circ$  - or from the left to the right. The image must also be rectangular. When rotating an image to an apparent solar  $270^\circ$  azimuth, crop a larger image is necessary. When the image is rotated, it may not be rectangular. With extra margins you can crop the rotated region of interest to a rectangular shape.



A larger original crop would not leave these empty regions after rotation

**Constraint - equal x-y scale in the image pixels**

**Are your camera pixels square or rectangular? AIP4WIN has a utility to square pixels.**

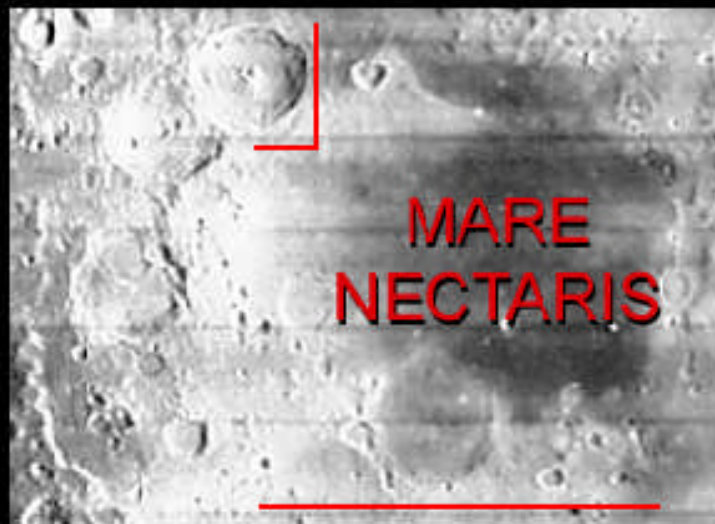


**Image photometry may be more effected by rotation of rectangular pixels as compared to square pixels.**

**Constraints - Foreshortening and distance from the apparent lunar center  
Equal spacing of x-y plane pixels or rectification**

The lunar-Lambert assumption, relied upon by the Carlotto algorithm, behaves very similar to the simple Lambert model near the centre of the lunar disk but strongly deviates from the Lambert model near the limb.

Foreshortening also invalidates the assumption that the linear distance between any two pixels in the horizontal x-y plane is the same.



**Mare Nectaris IV-100-M**



**Mare Nectaris IV-95-M foreshortened**

An empirical rule-of-thumb is that SfS only should be applied to features between  $\pm 30^\circ$  lunar latitude and  $\pm 30^\circ$  lunar longitude.

## **Constraint - Foreshortening and distance from the apparent lunar center**

**Foreshortening, within this feature constraint of lunar latitude and longitude, can be corrected pre-DEM processing or post-DEM processing.**

**Evans 2006a and Evans 2006b suggests two methods for adjusting for foreshortening. Neither method easily can be applied to an NxM pixel matrix. But this problem is inherent to modern astrophotographs that are matrices that are dimensioned by integer pixels.**

**In Evans 2006a, Evans suggests post-processing DEM heights with a tilt factor:**

$$\text{Tilt Angle} = 1/[\cos(b - b')*\cos(L-L')]$$

**In Evans 2006b, Evan suggests post-processing the DEM for separate North-South and East-West foreshortening adjustments:**

$$y_{NS} = 1 / \cos(b-b') \quad x_{EW} = 1 / \cos(L-L')$$

**where b, L are the feature lunar latitude and longitude and b', L' is the observer's lunar sub-Earth latitude and longitude.**

**Finally, Evans 2006b, suggests adapting the image to a cylindrical projection using the freeware IRIS.**

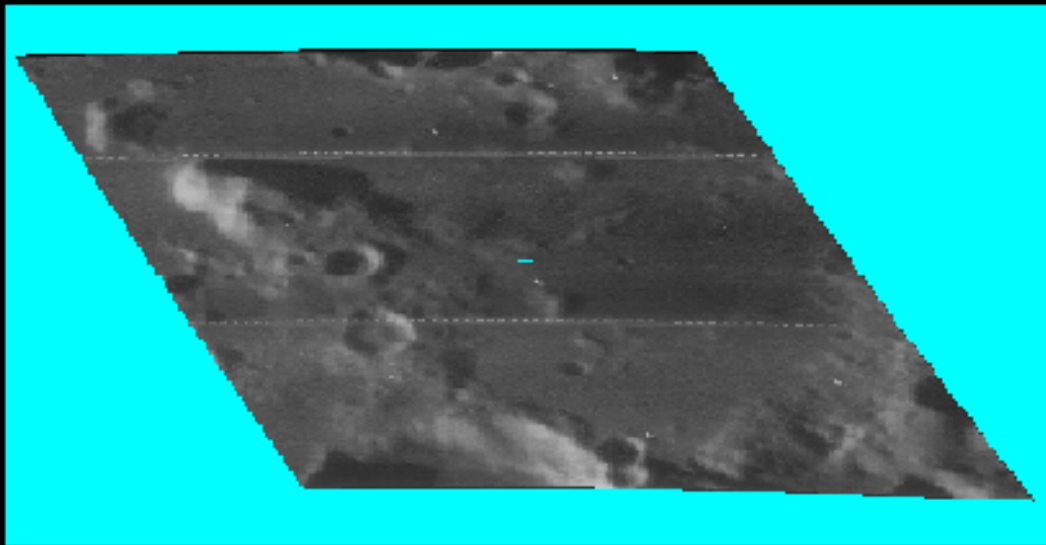
## **Constraint - Foreshortening and distance from the apparent lunar center**

**None of these solutions to the problem of foreshortening seems entirely satisfactory.**

**Another image processing solution, that became available after Evans 2006b, is Jim Mosher and Henrik Bondo's Lunar Terminator Visualization Tool (LTVT).**

**[http://inet.uni2.dk/~d120588/henrik/jim\\_ltv.html](http://inet.uni2.dk/~d120588/henrik/jim_ltv.html)**

**LTVT allows the user to register their images to a lunar sphere and then produce rectified images of the original.**



**However, LTVT's rectification algorithm are not fully stated in user documentation.**

**Image on lunar limb rectified in LTVT**

## Constraint(s) - Foreshortening and equal x-y image pixel scale

The spreadsheet presented here includes a simple calculator for adjusting a single point maximum elevation for foreshortening. Evans 2006a algorithm is used:

$$\text{Tilt Angle} = 1/[\cos(b - b')*\cos(L-L')]$$

The Evans 2006a tilt adjustment is accurate when applied to a single point - the maximum elevation in a DEM.

The second adjustment suggested in Evans 2006b -

$$y_{NS} = 1 / \cos(b-b') \quad x_{EW} = 1 / \cos(L-L')$$

- is not implemented because I believe it can only be applied as a preprocessing modification to the raw image. Changing the x-y scale to gradient relationship changes the resulting DEM's height estimate.

Evans 2006b method only works for images that are oriented to the lunar north pole. Here, images are rotated so the apparent solar azimuth is 270°.

Although the Evans 2006a tilt adjustment is provided for in the spreadsheet presented here, it is not recommended. The recommended solution for foreshortening is preprocessing rectification using LTVT.

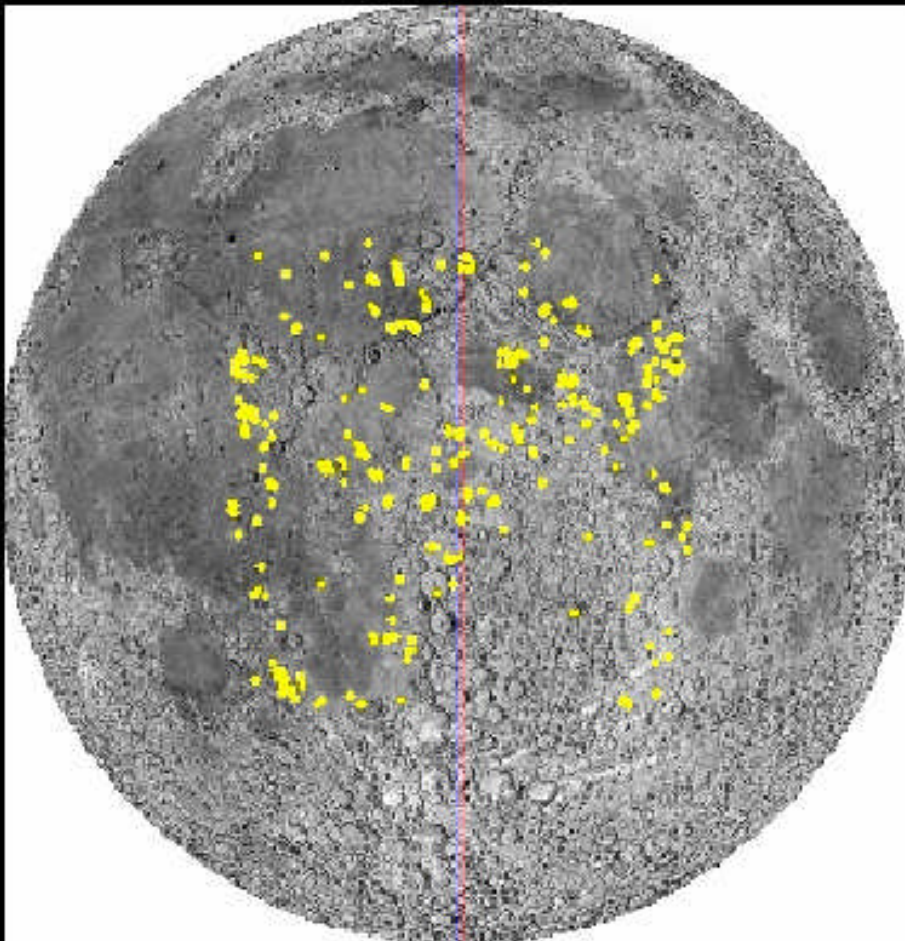
In the proposed LTVT rectification solution, before rotation, the raw image is rectified in LTVT. This adjusts x-y plane proportions for the feature's lunar latitude and longitude.

After pre-processing the image for rectification, SfS processing proceeds by the usual procedure.

## Constraint - Foreshortening and distance from the apparent lunar center

The constraint of restricting application of SfS to  $\pm 30^\circ$  lunar latitude and longitude does not translate into a small base catalogue of features. Taking only lunar domes in Kapral's and Garfinkle's 2005 dome catalogue, there are 396 domes meeting this criteria. This does not include additional domes that may meet the criteria due to libration.

This total does not include other features that may meet this position criteria, such as mare wrinkle ridges and faults.



Location of 396 verified and unverified domes in Kapral and Garfinkle's 2005 dome catalogue that meet the position criteria.

The supplemental materials to this presentation includes a table of the 396 verified and unverified domes.

Selenology Today is not the only journal source for lunar dome DEMs. Issues of ST have published DEMs for 16 domes.

Issue #1 - 13 domes, 13 DEMS  
Issue # 3 - 6 domes, 3 DEMS



## Constraint - Foreshortening and distance from the apparent lunar center

The most accurate DEMs are those made from images that require minimal rotation and rectification. The Moon's libration averages  $\pm 7.95^\circ$  in an east-west direction and  $\pm 6.85^\circ$  in a north-south direction (Westfall 2000).

Images that require a minor horizontal "flip" or minor rotation can be made by planning to image at times of most favorable illumination and/or libration.

Mosher's and Bondo's LTVT includes a solar angle planning tool. Once the date and time of a feature's favorable lighting is known, LTVT will seek all similar solar angles that will occur at an observing point over the next 12 months.

Date	UTC	Sun alt	Sun az	Moon alt
07/09/2007	11:27:06	4.7327	270.6651	41.835
11/20/2007	00:39:13	4.7327	92.0703	35.100

Sample data from LTVT predict feature showing next most favorable lightings of the Hortensius domes from author's observing point.

## **Constraint - Foreshortening and distance from the apparent lunar center**

**Planning for the most favorable imaging date and time is a two-step process:**

- 1) Find the best low-angle illuminations for the coming year when the Moon is at least  $30^\circ$  above the horizon and the Sun is beneath the horizon. This author uses a combination of Virtual Moon Atlas, LTVT and a personal planning spreadsheet to select promising dates.**
- 2) Amongst favorable lighting dates, find those dates with the best libration. This author uses the RASC Observer's Handbook to identify favorable librations.**

## Constraints - Processing time

In the spreadsheet presented here - DEMCarlottoMethod.xls - some coding includes a Big O recursive loop. As a result, processing times increase exponentially with image pixel dimensions:

Image dimensions (pixels)	Processing time
42 x 80	16 secs
70 x 100	1 minutes
225 x 225	37 minutes

Processing times in DEMCarlottoMethod.xls

## **Constraints - Implications for target selection**

**1) Uniform surface reflectance - Clementine false color ratio check**

**2) Uniform illumination**

**No dark wells or oversaturated bright surfaces**

**Height of feature limited to low heights and low slope angles**

**Solar angle of illumination - 4° to 10° best**

**3) Foreshortening and equal x-y pixel spacing**

**Within +/- 30 deg lunar latitude and longitude**

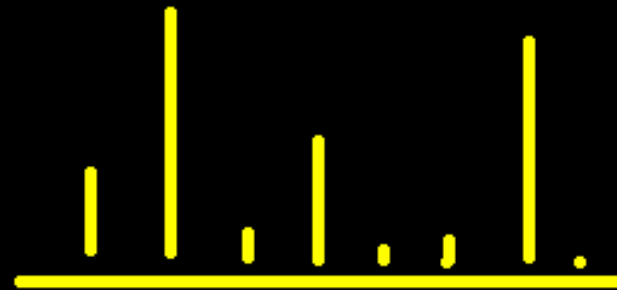
**Plan for best illumination and libration to minimize pre-processing rotation**

**4) Processing time - magnify feature so cropped image will be about 100x100 pixels in size**

## Review of key concepts

1) What basic nomenclature and terminology is used when discussing Shape-from Shading (SfS)?

reflectance or incidence map  
(brightness per pixel)



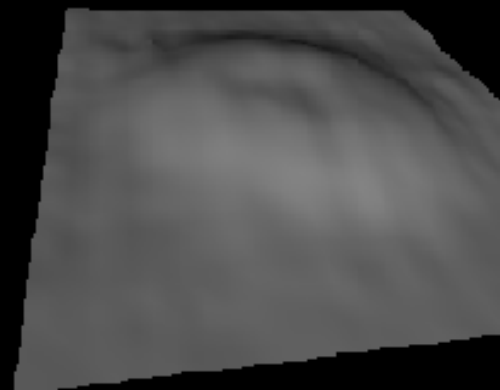
gradient map or map of slopes  
( z pixel / 1 pixel )



summed gradient map  
( z pixel / 1 pixel )



digital elevation model (DEM)  
( meters / pixels ) or  
( meters / meters )

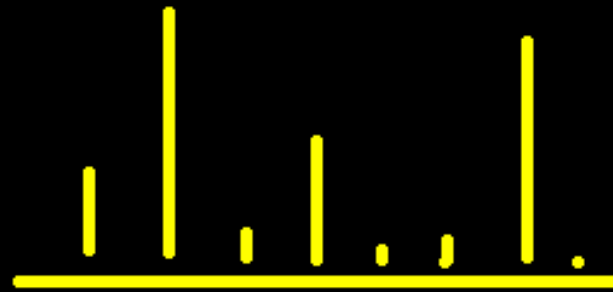


## Review of key concepts

2) How do simple SfS algorithms convert brightness to gradients and gradients to elevations in meters?

reflectance or incidence map  
(brightness per pixel)

conversion equation  
applied



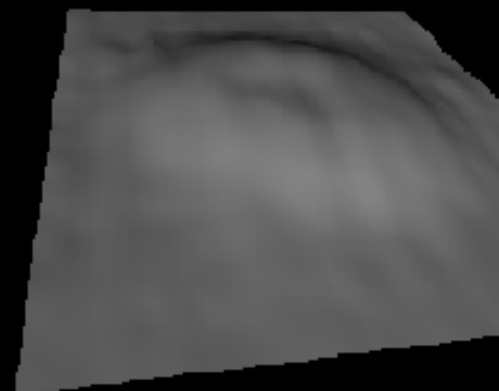
gradient map or map of slopes  
( z pixel / 1 pixel )

summation addition

summed gradient map  
( z pixel / 1 pixel )

multiply by  
horizontal scale

digital elevation model (DEM)  
( meters / pixels ) or  
( meters / meters )



## Review of key concepts

3) What SfS assumptions restrict the types of lunar features that it can be applied to?

- **Uniform surface reflectance - Clementine false color ratio check**
- **Uniform illumination**

**No dark wells or oversaturated bright surfaces**  
**Height of feature limited to low heights and low slope angles**

**Solar angle of illumination - 4° to 10° best**

- **Foreshortening and equal x-y pixel spacing**

**Within +/- 30 deg lunar latitude and longitude**

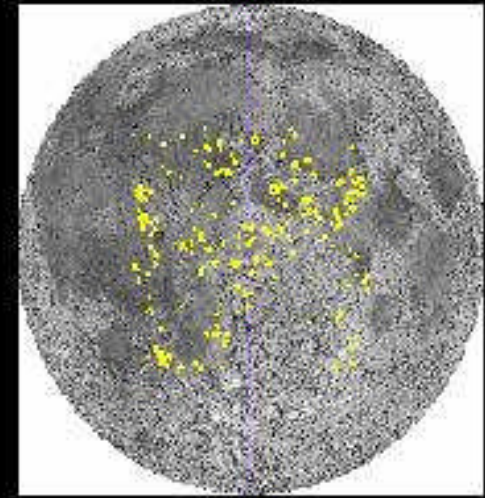
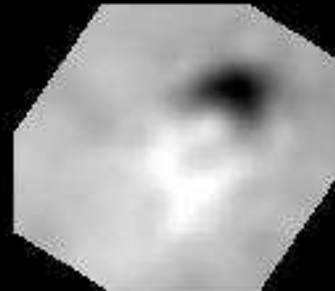
**Plan for best illumination and libration to minimize pre-processing rotation**

- **Processing time - magnify feature so cropped image will be about 100x100 pixels in size**

# End of Part 1 - Basic SfS Concepts



Katima, Alaska

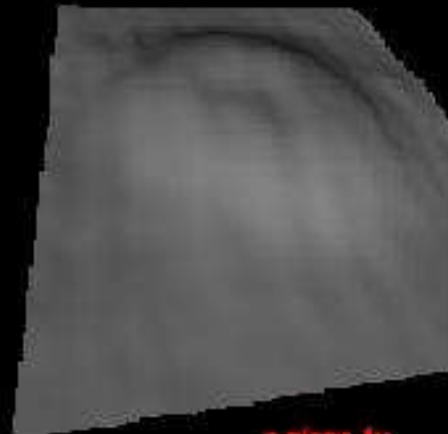


Mons Rumker - Apollo 16 AS16-07-13262



Martus Hills - Lunar Orbiter IV - N-15-42

3D floating point



z-plane 1c

